# The Creditor Channel of Liquidity Crises*

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**Abstract**

This paper presents a model to study the transmission of liquidity shocks across financial institutions through the creditor channel. In the model, a borrower institution obtains funds from a large institutional lender and small investors. When the large lender's asset market is hit by a liquidity shock, it might decide to withdraw funding extended to the borrower. The *potential* withdrawal by the large lender causes small investors to panic and to close positions even if the large lender does not. Facing funding problems, the borrower has to cut its activities, contributing to further shocks to the supply of market liquidity. The original shock is exacerbated, which reinforces withdrawals by all creditors. The model helps explain how the spreading of liquidity shocks from the broker-dealer sector to the hedge fund sector and the feedback contribute to a systemic crisis.

*JEL codes*: G01, G14, G21, G23, G24, D83, D53

Keywords: systemic crises, creditor runs, market liquidity, coordination risk, amplifications

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# 1 Introduction

Financial markets have undergone profound changes since the mid-eighties. Deregulation reshaped the financial landscape with the creation of new breeds of institutional investors. Innovation multiplied new financial contracts at a fast pace. Years of historically low interest rates, combined with new forms of debt financing, facilitated access to leverage. The increase in leverage happened simultaneously with changes in the composition of creditors, with institutional investors occupying a greater role in funding financial intermediaries. Now commercial banks source a large fraction of their funds from the wholesale market. Broker dealers rely heavily on counterparties for finance. Hedge funds depend a great deal on their prime brokers for lines of credit.

It is often argued that stronger relationships among financial institutions help the efficient dispersion of risk in the financial system. However, the financial crisis of 2007-2009 has shown that connections among financial players can also contribute to the contagion and amplification of risk, and to deepen a crisis. During the crisis, many hedge funds experienced severe liquidity shortages and had to drastically curb their trading. According to Mitchell and Pulvino (2012):

"The imminent failure of large Wall Street prime brokerage firms during the 2008 financial crisis caused a sudden and dramatic decrease in the amount of financial leverage afforded hedge funds... A primary consequence of this withdrawal of financing was the inability of hedge funds involved in relative-value trades to maintain prices of substantially similar assets at substantially similar prices." (pp. 469)

In response to the systemic fragility that erupted in the wholesale markets during both the 2007-2009 US financial crisis and the 2010-2012 EU sovereign and banking crisis, regulators have been considering actions that limit the use of wholesale funding by banks.[1] On their own initiative, banks have also been gradually moving towards a business model that relies on deposits and equity to a greater extent.

In this paper, we attempt to understand how liquidity shocks are transmitted via the creditor channel. More concretely, what is the exact chain of events and how does it contribute to a systemic crisis?

We build a stylized model in which a modern financial institution borrows in the interbank

[1] The Basel Committee and the FSA have proposals under discussion requiring banks to reduce their dependence on short-term wholesale funding.

market and from non-bank creditors. Specifically, we model a financial institution (e.g., a hedge fund), which obtains a significant fraction of its funds short term from a large financial institution (e.g., a prime broker), as well as small investors (e.g., client investors). The large lender is itself a leveraged institution, whose lending decisions depend on the strength of its balance sheet. The large lender invests in long-term illiquid assets (e.g., mortgage-backed securities) and lends short term to other institutions. Small investors do not face any balance sheet constraints, but each of them has less than perfect information about other investors' lending decisions.

We start by showing that in response to a large adverse shock to market liquidity, the large lender might withdraw its short-term loans to the borrower. Risk management that is concerned with mark-to-market losses in long-term illiquid assets, which negatively affect equity capital and margin requirements, makes the market-based lender hoard liquidity and reduce its exposure. The lender does this by cutting lending to other institutions, resulting in leverage that is procyclical, as evidenced by Adrian and Shin (2010).

We formally derive how the (potential) withdrawal by large lenders can then trigger two intertwining amplification mechanisms that lead to liquidity crunches for the borrower institution. First, the awareness that the large lender might be constrained and could *potentially* cuts its credit to the borrower can spark panic among spare-liquidity providers not facing balance sheet constraints. That is, some small investors will *preemptively* run on the borrower institution. More specifically, should the large lender decide to hoard liquidity, the borrower institution would be exposed to a sharp liquidity shortage. The borrower may then be forced to sell assets in illiquid markets at a loss. Aware of this possibility, some small investors will run preemptively. In other words, the potential action of the large creditor unnerves small investors and causes them to run. The possibility that the market-based large lender may soon become constrained thus threatening the liquidity status of the borrower is what triggers the run.

We believe that the above mechanism was in play in the 2007-2009 crisis when many financial institutions collapsed largely for liquidity reasons. According to some observers, institutions such as Northern Rock, Bear Stearns, Lehman Brothers and many hedge funds got into trouble not because they were ex-ante insolvent.[2] In many cases their capital reserves appeared ex-ante adequate. Even so, investors ran, worried that these financial institutions would be unable to roll over their short-term funding from wholesale markets and counterparties. Often, the massive withdrawals (run)

[2] See, e.g., Cox (2008), Gorton (2008), Brunnermeier (2009), Shin (2009).

started with *speculations* that major lenders were constrained and were going to cut their lending.

Second, the actions of squeezed borrower institutions can feedback to the lenders and fuel a downward spiral in liquidity. In our model, if the borrower trades in the large lender's asset market, a feedback loop arises: A negative shock to market liquidity causes the large lender's balance sheet to tighten, triggering lenders to cut funds to the borrower; this creates a funding problem for the borrower, which then becomes unable to perform its arbitrage activities; the result is a further deterioration in the liquidity of the assets held by the large lender. Because there is a coordination problem among multiple lenders, the feedback on liquidity interacts with the coordination problem.

We argue that both amplification mechanisms — the preemptive withdrawals by unconstrained investors and the pull back in liquidity provision by the squeezed borrower — are able to explain, to a large extent, the problems experienced by many hedge funds in 2007-2009. As many have argued, the crisis originated in the banking sector, and banks' troubles then imposed immense funding pressure on hedge funds. What we highlight in this paper is the role played by the two amplification effects. The potential withdrawal of bank credit caused hedge fund investors to panic, contributing decisively to the liquidity crunch of these funds.[3] The *Economist* wrote: "A fuller explanation must include the increasingly *jittery* nature of hedge funds' clients."[4] Ben-David et al. (2012) document that hedge fund selloffs were driven by both massive redemptions and margins calls. Mitchell and Pulvino (2012) show that the huge contraction in the hedge fund sector cannot be fully explained by bad assets. Additional evidence of our creditor channel mechanism can be found in the annual reports of major financial institutions. An excerpt from the 2008 annual report of Goldman Sachs states that: "Given the difficult market conditions, and in particular, the challenging liquidity and funding environment during 2008, we focused on reducing concentrated risk positions, including our exposure to leveraged loans and real estate-related loans."

An extensive literature has analyzed the amplification mechanisms of a liquidity shock.[5] Brunnermeier (2009) discusses various amplifying mechanisms in the context of the 2007-2009 crisis. What is new in our paper is that we explicitly model the role played by a large lender in amplifying the liquidity shocks. Specifically, our paper shows how two amplification mechanisms through the

[3] Although nominally characterized as equity partners, investors in hedge funds that are able to redeem at short notice have a payoff structure that resembles the one for a debtholder (see Brunnermeier (2009), Liu and Mello (2011), Shleifer and Vishny (1997)). Teo (2011) reports that the redemption frequency in hedge funds is very high.

[4] "Hedge Funds in Trouble: The Incredible Shrinking Funds", *Economist*, October 25-31 2008, pp. 87-88.

[5] See, e.g., the recent survey on financial crises and systemic risk by Brunnermeier and Oehmke (2013).

creditor channel contribute to severe liquidity shortages in financial institutions. First, we study a liquidity-based run. We show that a constrained large lender to a financial institution can potentially set off a self-fulfilling cycle of withdrawals by all lenders. The run is different from the pure self-fulfilling mechanism in Diamond and Dybvig (1983), where there is no trigger for a run. Second, we analyze a liquidity spiral. The spiral works through *the creditor channel*, complementing the 'margin spiral' in Brunnermeier and Pedersen (2009). In their model, market liquidity of the assets held by arbitrageurs directly impacts their ability to borrow (margin), i.e., *the borrower channel*. In other words, in their model the direction of the loop is: market liquidity → borrower → market liquidity; by contrast in our model the loop is: market liquidity → lenders → borrower → market liquidity. We show that the liquidity spiral which works through the creditor channel is reinforced by the coordination problem among lenders.

The paper is organized as follows. Section 2 presents the main model. Section 3 studies several extensions of the main model. Section 4 discusses the empirical implications. Section 5 concludes.

# 2 Model

In this section, we first present the model setup, then solve the equilibrium, and finally analyze the implications of the model.

## 2.1 Model setup

Consider an economy with three types of agents: financial institution H (hereafter FI-H, e.g., a hedge fund), financial institution B (FI-B, e.g., a broker), and "small" investors.[6] FI-H is funded by its own capital and the borrowing from FI-B and small investors. FI-B is the single "large" lender to FI-H, and its loans represent a proportion $\lambda$ of FI-H's total debt.[7] The remaining proportion, $1 - \lambda$, of FI-H's debt comes from a continuum of small lenders. All debt is of the same seniority; this assumption will be relaxed later in the extended model in Section 3. Figure 1 summarizes the

[6] In our model, we could have explicitly used a general-equilibrium framework with a continuum of identical agents of each type. However, this would complicate the techniques in computing the equilibrium. In fact, in our global game setup where each borrower has a continuum of small lenders, solving this equilibrium itself involves technical complications. A simplified setup with an 'aggregate' agent for each type is therefore used.

[7] For simplicity, we assume that there is only one large lender, but the insight applies when there are multiple large lenders.

balance sheet linkages among different financial institutions.

The setup captures the feature that modern financial institutions borrow in the interbank market and from non-bank creditors. FI-H can be interpreted as a hedge fund, and FI-B as its prime broker. FI-H could also be a commercial bank or an investment bank that relies heavily on the wholesale market or counterparties for borrowing; in this case, FI-B would be the wholesale market lender or counterparty.[8] Small lenders in our model can be interpreted as non-bank creditors, who are spare-liquidity providers to the financial system with no balance sheet constraints.

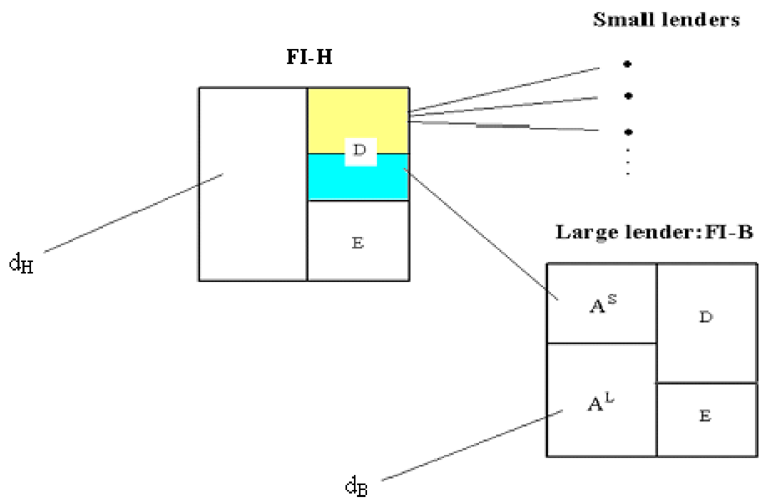


**Figure 1**: Balance sheet linkages

The model has three dates: $T_0$, $T_1$ and $T_2$. Later, we split $T_1$ into $T_1$ and $T_{1+}$. All three types of agents are risk-neutral. The risk-free rate is assumed to be zero.

#### 2.1.1 Large lender: FI-B

At $T_0$, FI-B has two types of assets on its balance sheet: short-term assets ($A^S$) and long-term assets ($A^L$). These assets are financed with debt ($D$) and equity ($E$). That is, FI-B is a *leveraged* financial institution.

We assume that FI-B's lending to FI-H is short term.[9] In our model, short term means that

[8] In recent years, all types of financial institutions (commercial banks, investment banks, hedge funds, insurance companies) are involved in wholesale lending. One example is the creditor structure of off-balance-sheet vehicles, SIVs, which are essentially unregulated departments of banks. Lenders to SIVs include commercial banks, investment banks and hedge funds, each investing in a different tranche.

[9] See, e.g., the evidence in Afonso, Kovner and Schoar (2013).

FI-B has the right to call the loan at $T_1$ or extend it until $T_2$. For simplicity, we assume that FI-B holds one unit of the long-term asset, denoted by $CDO$. The fundamental value of $CDO$ at $T_0$ is a constant $v$, which equals its expected liquidation value at maturity $T_2$. At $T_0$, many investors including FI-B hold $CDO$. However, for various reasons, some investors may decide to sell some units of $CDO$ before maturity $T_2$. We call these investors noise or liquidity traders. Imperfect market depth translates into a cost to early liquidation. The cost is reflected in a downward-sloping demand curve for $CDO$:

$$p = v - d_B \cdot s, \tag{1}$$

where $d_B$ is a measure of market depth which can be time-varying, $s$ is the aggregate quantity of asset $CDO$ that investors sell, and $p$ is the price.[10]

Liquidity/noise traders trade at any time before $T_2$. For simplicity and without loss of generality, we assume that they trade at $T_{1+}$, where $T_{1+}$ is an instant after $T_1$. We assume that the ex-ante aggregate demand for liquidity from these investors is normally distributed with $\widetilde{s} \sim N(0, \sigma^2)$. Hence, the mark-to-market value of FI-B's long-term asset at $T_{1+}$, denoted by $A^L_{1+}$, is distributed according to

$$A^L_{1+} \sim N(v, \sigma^2 d_B^2). \tag{2}$$

Also, the balance sheet equation $A^S + A^L_t = D + E_t$ is always valid at any time $t$, where $A^L_t$ and $E_t$ are the mark-to-market values of the long-term asset and equity, respectively.

FI-B manages risk with tools such as Value at Risk (VaR) and takes prudent and precautionary actions toward unfolding events (see, e.g., Adrian and Shin (2012) for the micro-foundation reason for using VaR). Specifically, we assume that FI-B always requires that the probability of its mark-to-market leverage (in terms of the asset-to-equity ratio) exceeding an upper limit $\overline{L}(> 1)$ be less than a small percentage $\alpha$ (e.g., $\alpha = 5\% < \frac{1}{2}$). The reason for this precautionary rule is that financial institutions can face severe consequences such as margin calls, debt downgrading and difficulty in refinancing if their leverage ratios exceed certain limits.[11] In an attempt to manage

[10] This is a similar specification to those in Grossman and Miller (1988), Campbell, Grossman and Wang (1993), Morris and Shin (2004), and Brunnermeier and Pedersen (2005).

[11] If the mark-to-market value of an asset goes down, so does its collateral value to the creditors, which is derived from the resale value. Hence, lower market liquidity increases the probability of margin calls. Margin calls at $T_{1+}$ are also very costly because long-term illiquid assets will go at fire sale prices, given that short-term loans can only be called at $T_2$ if they had not been at $T_1$.

the risk of such costly events, FI-B is assumed to take prudent actions *beforehand*.

In our model, the adverse event — the risk of excessive leverage caused by fluctuations in the market price of $CDO$ — occurs at $T_{1+}$. Therefore, FI-B's risk management requires that the following condition is satisfied at $T_1$:

$$\Pr(\frac{D+E_{1+}}{E_{1+}} > \overline{L}|\digamma_1) < \alpha, \tag{3}$$

where $\digamma_1$ is the information set of FI-B at $T_1$.[12] The strength of the balance sheet can be expressed in terms of leverage; in fact, there is a one-to-one mapping between the pair $(D, A^S)$ and the pair $(E_0,\, L_0)$: $\begin{cases} D = E_0(L_0 - 1) \\ A^S = E_0 L_0 - v \end{cases}$, where $E_0$ is the equity value and $L_0$ is the asset-to-equity ratio at $T_0$. Denote the $\alpha$-percentile of $A^L_{1+} \sim N(v, \sigma^2 d_B^2)$ by $C$, i.e., $\alpha \equiv \Phi(\frac{C-v}{\sigma d_B})$.

We assume that at $T_1$ FI-B receives *perfect* information about the market depth, $d_B$. If $d_B$ happens to be high, FI-B may need to deleverage by calling its short-term loans $A^S$ to comply with its risk management requirement, (3).[13] Lemma 1 follows.

**Lemma 1** *After FI-B has received information about market depth $d_B$, its decision rule at $T_1$ based on its risk management requirement is*

$$(L_0, d_B) \longmapsto \begin{cases} Call & L_0 \geq L_B^*(d_B) \\ Hold & L_0 < L_B^*(d_B) \end{cases},$$

*where*

$$L_B^*(d_B) \equiv \overline{L} - \frac{v - C(d_B)}{E_0}(\overline{L} - 1) \tag{4}$$

*is a threshold and a decreasing function of $d_B$.*

***Proof.*** *See Appendix.* ■

Lemma 1 says that for a given $L_0$, when FI-B receives information of a sufficiently high $d_B$ at $T_1$ such that $L_0 \geq L_B^*(d_B)$ , it cuts its short-term lending $A^S$; otherwise, it extends its loans. In

[12] Without loss of generality, we assume this constraint is not binding and is slack at $T_0$.

[13] Without loss of generality, we assume that FI-B either calls its entire loans or does not call at all. Essentially, we emphasize the *possibility* rather than the *necessity*. That is, what matters for our purpose is that there is a sufficiently large negative shock to $d_B$ such that FI-B calls a large amount of its loans. When the shock to $d_B$ is small enough, FI-B may not call a large amount of its loans, which, nevertheless, is not what we are interested in. The same effect could also be achieved in our model if $d_B$ followed a discrete stochastic process with jumps, so that a shock to $d_B$ would cause FI-B to call a block size of the loans.

other words, an increase in $d_B$ can lead to a reduction in $A^S$. The intuition behind Lemma 1 is as follows. As market liquidity shrinks (i.e., $d_B$ increases), the value of the long-term assets becomes more volatile (see (2)) and the distribution becomes more fat-tailed (i.e., both upside and downside risks increase). The risk management (VaR) only cares about the downside risk. So the loss in asset values translates into an increase in the asset-to-equity ratio; that is, the mark-to-market leverage increases. In order to bound the potential leverage to a certain level, the lender must start deleveraging and liquidate positions, first and foremost by liquidating relatively liquid assets such as loans extended to other institutions.[14]

### 2.1.2 Borrower financial institution: FI-H

FI-H's asset is normalized to one unit of the long-term asset with random payoff $\widetilde{X}$ at $T_2$. We denote the fundamental value of FI-H's long-term asset by $f = \mathbb{E}(\widetilde{X})$, i.e., the asset's expected liquidation value at $T_2$ perceived at $T_1$.[15] The asset is illiquid in the sense that a premature sale at $T_1$ results in a downward-sloping sale price of $p = f - d_H \cdot s$, where $d_H$ is the market depth and $s$ ($\in [0, 1]$) represents the units of selling from FI-H. Basically, we think of FI-H as representing the 'aggregate' financial institution of this type, and FI-H's sales impact the market price (see also Stein (2012)). FI-H's revenue from selling quantity $s$ equals $\pi(s) = ps = (f - d_H \cdot s) \cdot s$.[16]

FI-H has debt with face value $K$. The debt is short term, and the debtholders have the right to call their loan at $T_1$, in which case FI-H is liable to repay them at face value. If the debtholders do not call their lending at $T_1$, their lending is automatically extended until $T_2$, in which case FI-H is required to repay the debtholders with interest; the total *notional* amount of repayment at $T_2$ is $KR$, where $R$ is the gross interest rate ($R > 1$). A proportion $\lambda$ of FI-H's debt is held by FI-B, and is identical to FI-B's short-term assets $A^S$; and $1 - \lambda$ is held by a continuum of small lenders.

As in Diamond and Dybvig (1983), we assume further that if the debtholders call their loans

[14] That it is optimal for the large lender to liquidate liquid assets first has been asserted by Scholes (2000, pp. 19): "In an unfolding crisis, most market participants respond by liquidating their most liquid investments first to reduce exposures and to reduce leverage." The price reduction of the long-term asset (at $T_1$) resulting from the loss of liquidity is likely to be temporary. Selling the asset would realize an actual loss. Empirically, the 2007-2009 financial crisis witnessed that financial institutions held on to their long-term illiquid assets until the governments bailed them out.

[15] At $T_1$, FI-H's lenders have homogeneous information and beliefs about the distribution of $\widetilde{X}$.

[16] The bank-run game works as long as there is a liquidation discount in asset sales.

at time $T_1$, the only way that FI-H can repay is to liquidate assets. It cannot find new creditors that are ready to replace the old creditors. This assumption can be justified by the literature on relationship lending.[17] The idea is that it takes time for a creditor to build firm-specific lending relationships with a borrower, either because of moral hazard related to monitoring the borrower, or because of adverse selection related to the quality of the borrower. Mitchell and Pulvino (2012) present empirical evidence supporting this assumption for broker lending to hedge funds. Huang and Ratnovski (2011) analyze the role of wholesale market lenders in monitoring banks.

Although it is suboptimal for FI-H to sell any unit of asset at $T_1$ (because of liquidation discount in selling), it might be forced to do so because its creditors may decide not to extend their short-term loans.

We now state the payoffs of a creditor when it decides either to call the loans at time $T_1$ (and thus forego the interest), or to extend the loans until time $T_2$. The payoff is a function of the aggregate number of creditors calling. Denote by $u$ the proportion of creditors that call the loans at $T_1$, where $0 \leq u \leq 1$. Lemma 2 states the (local) *strategic-complementarity* payoff structure.[18]

**Lemma 2** *The payoff function for an individual creditor of calling the loan at $T_1$ is* $w^C(u) = \begin{cases} K & \text{if } 0 \leq u \leq \frac{f-d_H}{K} \\ \frac{f-d_H}{u} & \text{if } \frac{f-d_H}{K} < u \leq 1 \end{cases}$*, while the payoff function of extending the loan until $T_2$ is*

$$w^H(u) = \begin{cases} E[\min(\frac{(2d_H-f)+\sqrt{f^2-4u\,\mathrm{K}d_H}}{2(1-u)d_H}\widetilde{X}, KR)] & \text{if } 0 \leq u \leq \frac{f-d_H}{K} \\ 0 & \text{if } \frac{f-d_H}{K} < u \leq 1 \end{cases}.$$

***Proof.*** *See Appendix.* ■

We also denote by $\triangle w(u)$ the difference in payoffs between holding and calling the loan at $T_1$, as a function of $u$. That is,

$$\triangle w(u) = w^H(u) - w^C(u) = \begin{cases} E[\min(\frac{(2d_H-f)+\sqrt{f^2-4u\,\mathrm{K}d_H}}{2(1-u)d_H}\widetilde{X}, KR)] - K & \text{if } 0 \leq u \leq \frac{f-d_H}{K} \\ -\frac{f-d_H}{u} & \text{if } \frac{f-d_H}{K} < u \leq 1 \end{cases}.$$

Figures 2(a) and 2(b) depict the payoff functions (where $\widetilde{X}$ follows a two-state distribution).[19]

[17] See, e.g., Calomiris and Kahn (1991) and Diamond and Rajan (2000, 2001).

[18] Local strategic complementarities here mean that the payoff difference function of holding versus calling is not necessarily monotonic in the total size of calling, but satisfies the "single crossing property" as in Goldstein and Pauzner (2005).

[19] For simplicity, we can assume that $\widetilde{X}$ follows a two-state distribution $\widetilde{X} \in \{0, \bar{X}\}$, and the probability of realizing cash flow $\bar{X}$ at $T_2$ perceived at $T_1$ is $\theta$.

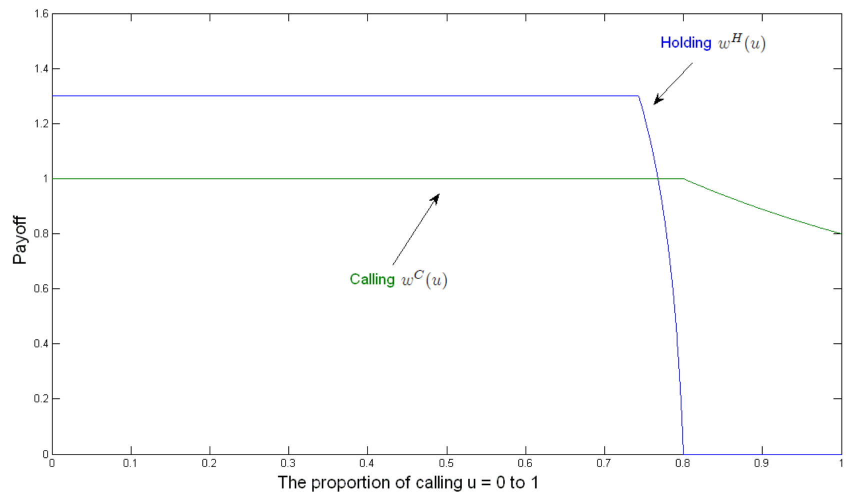


**Figure 2(a)**: Payoff functions of holding and calling

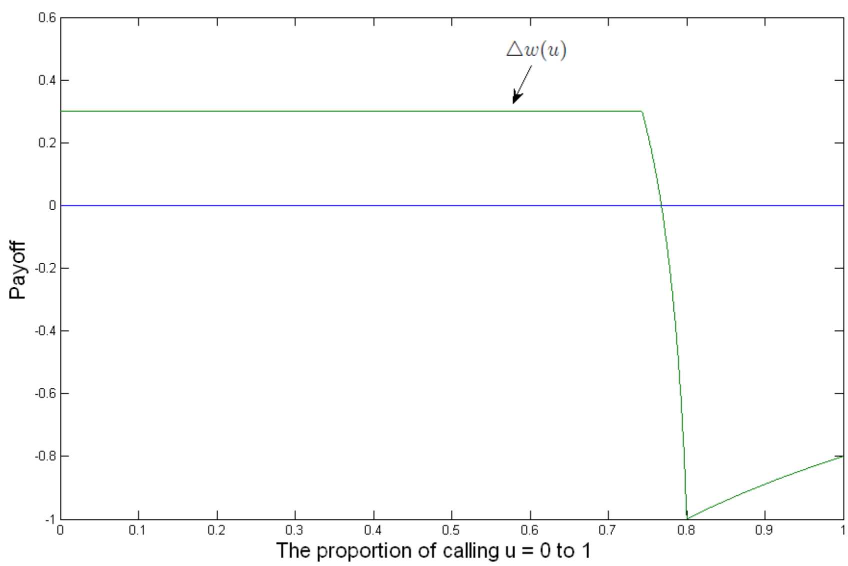


**Figure 2(b)**: Difference in payoff between holding and calling

From Figure 2(a), we can see the (local) strategic-complementarity payoff structure: If a high proportion (a high $u$) of creditors call, the optimal strategy for an individual creditor is also to call. If a low proportion (a low $u$) of creditors call, the optimal strategy for an individual creditor is also to hold (that is, not call).

### 2.1.3 Small lenders

Small lenders in our model should be interpreted in a broad sense. They are not only typical debtholders. If FI-H is a hedge fund, then small lenders can also be interpreted as the hedge fund's limited partners, such as high-net-worth individuals and funds of hedge funds, as long as these are allowed to redeem their investments early (Shleifer and Vishny (1997)). In fact, many hedge funds have, well before the 2007-2009 crisis, considerably reduced the lock-up period. Teo (2011) reports that 60% of the funds in his sample (or 5,015 funds) allow for monthly or more frequent redemptions, and 32% (or 2676 funds) allow for redemptions every one to three months. Although nominally characterized as equity partners, investors in funds that are redeemable at short notice have a payoff structure similar to that of a debtholder. Brunnermeier (2009), Chen, Goldstein and Jiang (2010), and Liu and Mello (2011) provide detailed arguments showing why a first-mover advantage can make financial institutions in general, and not just banks, subject to runs. We will explicitly study the small investors as equity investors in the extended model in Section 3.

Unlike the large lender FI-B, small lenders are not themselves subject to any balance sheet constraints.

#### 2.1.4 Information, decisions, and timeline

At $T_1$, both the large lender FI-B and the small lenders receive *perfect* information about market depth $d_H$ and $d_B$.[20] However, the strength of FI-B's balance sheet is not public information. Only FI-B itself knows $L_0$. Other investors receive signals about $L_0$.

It is realistic to assume that information about a balance sheet, valued at market prices, is non-public. First, the mark-to-market value changes frequently, and adjusting book value for impairments requires information that is not easily gathered by outsiders. Second, financial institutions differ significantly from non-financial firms in that their capital structure and financial positions can be quickly altered by trading and risk management as well as by financial commitments. Events after the collapse of Lehman Brothers in September 2008 have shown that a primary difficulty in financial markets was that nobody had a clue about the true strength of the balance sheet of big financial players.

[20] We assume that small lenders also observe $d_B$. A weaker assumption that small lenders receive imperfect information (signals) about $d_B$ does not change the model results. In reality, small investors can learn about market depth by observing spreads and other material trading information. Investors can obtain a lot of information about market liquidity and depth even if they are not experts on that market. Later, we will also consider the case where $d_H$ and $d_B$ are perfectly correlated, under which the assumption is naturally true.

Specifically, we assume that small lender $i$'s signal about the leverage of FI-B is given by

$$L_0^i = L_0 + \epsilon^i,$$

where $\epsilon^i$ is uniformly distributed with support $[-\epsilon, \epsilon]$, and $\epsilon^i$ is independent from $\epsilon^j$ for $i \neq j$. Furthermore, as in the global games literature, we assume that $L_0$ has an improper prior over the real line.

At $T_1$, the large lender FI-B and the small lenders decide *simultaneously* whether to call the loans. The large lender FI-B's strategy at $T_1$ is a map

$$(L_0, d_H, d_B) \longmapsto (\text{Call, Hold}),$$

while each small lender $i$'s strategy at $T_1$ is a map

$$(L_0^i, d_H, d_B) \longmapsto (\text{Call, Hold}).$$

Figure 3 describes the timeline.

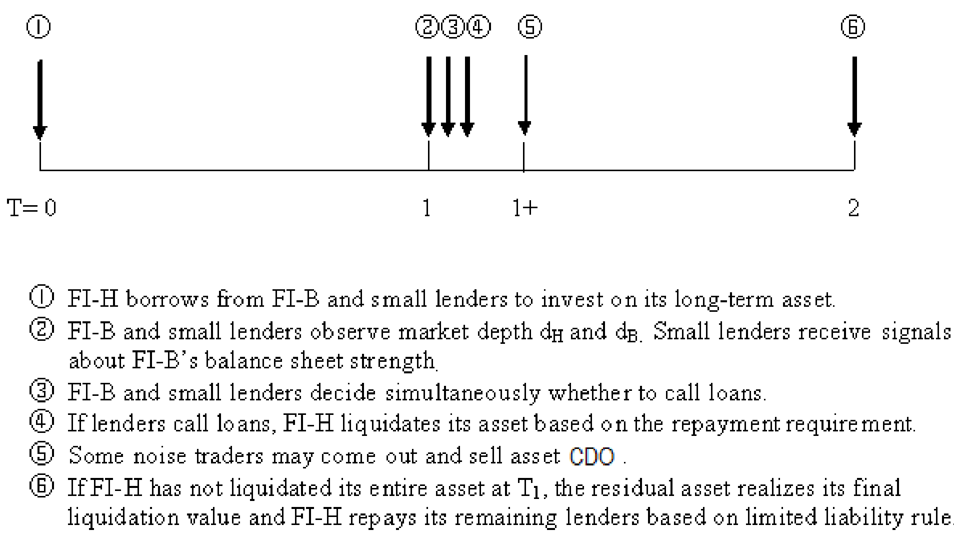


**Figure 3**: Timeline of events

## 2.2 Equilibrium

We are interested in the threshold equilibrium, where every lender uses a threshold strategy.[21] The strategy of the small lenders is

$$(L_0^i, d_H, d_B) \longmapsto \begin{cases} \text{Call} & L_0^i \geq L_S^*(d_H, d_B) \\ \text{Hold} & L_0^i < L_S^*(d_H, d_B) \end{cases},$$

where $L_S^*(d_H, d_B)$ is the threshold, while the strategy of the large lender is

$$(L_0, d_H, d_B) \longmapsto \begin{cases} \text{Call} & L_0 \geq L_B^*(d_H, d_B) \\ \text{Hold} & L_0 < L_B^*(d_H, d_B) \end{cases},$$

where $L_B^*(d_H, d_B)$ is the threshold.

### 2.2.1 Equilibrium when asset markets H and B are independent: Amplification 1

In this equilibrium, we assume that asset markets H and B are independent. More specifically, $d_H$ and $d_B$ are independent. This way we abstract away from the feedback loop between the two markets and focus on other aspects of the model.

Figure 4 illustrates the idea of the benchmark equilibrium. Each small lender's decision depends on its beliefs about the large lender's action, as well as on the actions of other small lenders. The large lender FI-B's decision depends on whether its risk management constraint is binding. However, we need to consider an *endogenous* feedback. The large lender's potential withdrawal triggers some small lenders to run, which in turn precipitates the large lender's withdrawal due to the strategic complementarities in payoffs. Therefore, FI-B's decision depends on both its risk management requirement and its beliefs about the actions of small lenders.

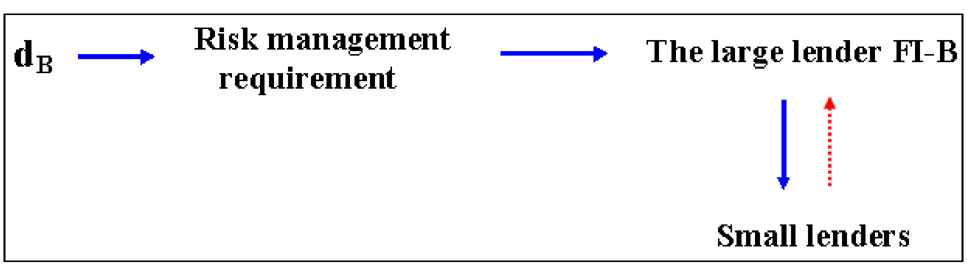


**Figure 4**: Idea of equilibrium

[21] In the financial economics literature on applications of global games, the threshold equilibrium is of primary interest. For example, Morris and Shin (2004, 2009) and He and Xiong (2012) consider only threshold equilibria.

An interesting result of our model is that we prove that in equilibrium the feedback described above does not occur. That is, FI-B's decision is solely determined by its own risk management concerns. Intuitively, because the large lender is big (and thus the aggregate size of small lenders is small), that some small lenders call is not sufficient to precipitate the large lender into calling. This result is also convenient for our analysis because such feedback is not a focus of our paper.[22]

We find the threshold equilibrium in two steps.

**Step 1:** Suppose that FI-B uses the threshold strategy in Lemma 1. Then find the equilibrium strategy among small lenders.

**Step 2:** Given the small lenders' strategy in Step 1, prove that FI-B indeed uses the strategy in Lemma 1.

Proposition 1 gives the result of the first step.

**Proposition 1** *When $\lambda$ is sufficiently high such that $\lambda \geq \underline{\lambda}(K, R, f, d_H)$, where $\underline{\lambda}$ is non-negative and solves $\int_0^{\lambda^T - \underline{\lambda}} \triangle w(u)du + \int_{\lambda^T}^{1} \triangle w(u)du = 0$ or $\int_0^{\lambda^T} \triangle w(u)du + \int_{\lambda^T + \underline{\lambda}}^{1} \triangle w(u)du = 0$ and $\lambda^T$ is a cutoff defined by $\triangle w(\lambda^T) = 0$, there exists a unique threshold equilibrium among the small lenders given the large lender's threshold strategy in Lemma 1:*

$$L_S^*(d_H, d_B) = L_B^*(d_B) + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m(K, R, f, d_H, \lambda), \tag{5}$$

*where $m$ solves the equation $\int_0^{m} \triangle w(u)du + \int_{m+\lambda}^{1} \triangle w(u)du = 0.$*

***Proof.*** *See Appendix.* ■

The following corollary immediately results from Proposition 1.

**Corollary 1** *No matter what threshold strategy the large lender uses, in equilibrium there is a proportion, $m$, of the small lenders that run before the larger lender.*

***Proof.*** *See Appendix.* ■

In equilibrium, the total proportion of investors withdrawing is given by

$$u(L_0) = \begin{cases} 0 & L_0 < L_S^* - \epsilon \\ (1-\lambda)\frac{L_0 + \epsilon - L_S^*}{2\epsilon} & L_S^* - \epsilon \leq L_0 < L_B^* \\ \lambda + (1-\lambda)\frac{L_0 + \epsilon - L_S^*}{2\epsilon} & L_B^* \leq L_0 \leq L_S^* + \epsilon \\ 1 & L_0 > L_S^* + \epsilon \end{cases}, \tag{6}$$

[22] Corsetti et al. (2004) focus on studying such feedback.

which is depicted by Figure 5. FI-B withdraws at the threshold $L_0 = L_B^*(d_B)$. Some small investors begin to withdraw at $L_0 < L_B^*(d_B)$, before the large lender decides to withdraw. In fact, there is altogether a proportion $m$ of the small lenders that run before FI-B, where $m$ is endogenously given in our model. Importantly, *$m$ is independent of the threshold $L_B^*$*. That is, no matter what threshold strategy the large lender uses, given the large lender's threshold strategy, the equilibrium among the small lenders results in a proportion $m$ of them withdrawing before the large lender does. This result will play an important role when we consider the full equilibrium in the next subsection.

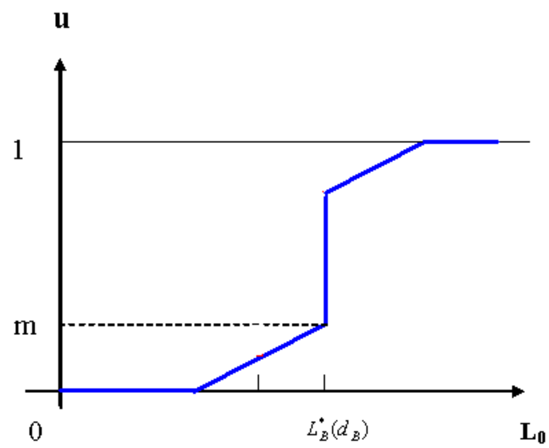


**Figure 5**: Total proportion of lenders withdrawing in equilibrium

In the second step, we show that the large lender's best response to the small lenders' strategy is to use the strategy in Lemma 1.

**Proposition 2** *When $\lambda \geq \underline{\lambda}(K, R, f, d_H)$, given that the small lenders use the strategy in Proposition 1, the large lender's best response is to call its loan if and only if $L_0 \geq L_B^*(d_B)$ as in Lemma 1. That is, the large lender's withdrawal decision coincides with its risk management requirement.*

***Proof.*** *See Appendix.* ■

The intuition behind Proposition 2 is as follows. Because the large lender can potentially withdraw, some small lenders withdraw first. Yet, that a low proportion of the small lenders withdraw (i.e., in equilibrium the proportion is $m$, which is lower than $\lambda^T$) is not sufficient to cause a feedback to the large lender. So the large lender's decision is determined only by its own risk management requirement considerations. We should emphasize that the condition of no feedback is not strong. In fact, as long as a unique threshold equilibrium exists among the small lenders, there is no feedback. That is, a sufficient condition for no feedback is $\lambda \geq \underline{\lambda}$, which is weaker than $1 - \lambda \leq \lambda^T$.

### 2.2.2 Equilibrium when asset markets H and B are dependent: Amplification 2

In this subsection we model the feedback from FI-H to $d_B$. Suppose that FI-H represents investors with market expertise, such as hedge funds. The social role of these sophisticated investors is to trade and hold illiquid assets to provide market liquidity. When these investors face funding constraints and are forced to unwind positions, market liquidity is reduced. A downward spiral then arises: The fact that FI-B may withdraw induces some small investors to run. The run by the small lenders causes FI-H to reduce its exposure and limit arbitrage activities. The reduction of arbitrage worsens *market liquidity* $d_B$ further. Lower market liquidity, in turn, precipitates FI-B into withdrawing. Figure 6 illustrates the feedback loop.

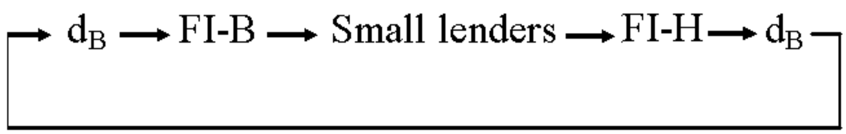


**Figure 6**: Feedback loop

We assume that the illiquid assets of FI-H and FI-B are in a similar category. Alternatively, investors in asset markets H and B face the *same* group of risk-averse market-makers. Market-makers have a limited capability to absorb risky assets because of their risk-aversion utility. If investors in market H unload some assets and sell them to the market-makers, the market-makers will be less able to buy more risky assets, including assets from market B. That is, asset sales in market H will reduce market liquidity in market B.

More concretely, we assume that the market-makers in markets H and B are the same, and $d_H$ and $d_B$ are identical. At an extreme, FI-H and FI-B may hold the same asset, say mortgage-backed securities (MBS). Suppose investors in market H unload $s_1$ amount of the asset *first*, and investors in market B unload $s_2$ amount *later*. Then, the demand curve faced by the sellers in market B is

$$p = v - d_B \cdot (s_1 + s_2),$$

rather than $p = v - d_B \cdot s_2$. Therefore, the market depth from the perspective of the sellers in market B is $d_B^{'} = \frac{v-p}{s_2} = \frac{s_1+s_2}{s_2} d_B$ rather than $d_B$; clearly, $d_B^{'} > d_B$. That is, the sales in market H reduce market liquidity in market B.[23]

Now we consider the interaction between the large lender FI-B and the small lenders. FI-B's decision has a *direct* impact on small lenders' decisions because of the strategic complementarities in payoffs. The small lenders' decision *indirectly* impacts the decision of FI-B because their withdrawals force FI-H to conduct fire sales, which in turn impacts the degree of market liquidity $d_B$. The equilibrium is the solution to a fixed-point problem. Conveniently, from (5), we know that the small lenders have a *linear* response to FI-B's strategy. That is, whatever the threshold of the large lender, given its threshold $L_B^*$, the small lenders' response is to use a threshold $L_S^* = L_B^* + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m(K,\, R,\, f,\, d_H,\, \lambda)$. Therefore, we only need to find $L_B^*$ in equilibrium.

By Corollary 1, there is a proportion $m$ of small lenders that run on FI-H before the large lender FI-B does. That is, ex post, in equilibrium, when the realization is $L_0 = L_B^*$,[24] there is a proportion $m$ of small lenders that withdraw. This means that FI-H needs to sell $s_1$ units of the asset at $T_1$ to repay the small lenders, just before FI-B starts calling its loans, where $s_1 = \frac{f-\sqrt{f^2-4m\,\mathrm{K}d_H}}{2d_H}$.[25] Hence, at $T_{1+}$, the mark-to-market price of $CDO$, in contrast with (1), is

$$p = v - d_B \cdot (s_1 + \widetilde{s}),$$

where $s_1$ is the sales by FI-H at $T_1$ and $\widetilde{s}$ is the sales by noise traders at $T_{1+}$. Equivalently, although FI-B observes at $T_1$ a market liquidity of $d_B$, it *rationally anticipates* the realized market liquidity (for investors in market B) at $T_{1+}$ of $d_B^{'} = \frac{s_1+\widetilde{s}}{\widetilde{s}} d_B$. Thus, the mark-to-market value of FI-B's long-term asset at $T_{1+}$ is distributed according to $A_{1+}^L \sim N(v - d_B \cdot s_1,\, \sigma^2 d_B^2)$.

In Lemma 1, we have shown that if the asset value follows the distribution $A_{1+}^L \sim N(v, \sigma^2 d_B^2)$, then FI-B's threshold, which results from its risk management requirement, is $L_B^* = \overline{L} - \frac{v-C(d_B)}{E_0} \cdot (\overline{L} - 1)$. Now the asset value has the distribution $A_{1+}^L \sim N(v - d_B \cdot s_1,\, \sigma^2 d_B^2)$. Denoting by $C^{'}$ the $\alpha$-percentile of the new distribution of $A_{1+}^L$, i.e., $\alpha \equiv \Psi(\frac{C^{'}-v+d_B \cdot s_1}{\sigma d_B})$, we have the revised threshold $L_B^{*'} = \overline{L} - \frac{v-C^{'}(d_B)}{E_0} \cdot (\overline{L} - 1)$, which is certainly lower than the original threshold $L_B^*$ because $C^{'}(d_B) < C(d_B)$.

**Proposition 3** *If asset markets H and B are co-dependent, a unique threshold equilibrium of the*

[23] As in Brunnermeier and Pedersen (2009), market liquidity is measured as the degree to which the market price of an asset is depressed away from its 'fundamental' value.

[24] Rigorously, it should be $L_0 = L_B^*$-, where $L_B^*$- is lower than but infinitely close to $L_B^*$.

[25] Recall the calculation of $s$ in the proof of Lemma 2.

*model exists, where the large lender FI-B uses the threshold*

$$L_B^{*'} = \overline{L} - \frac{v - C^{'}(d_B)}{E_0} \cdot (\overline{L} - 1),$$

*which is lower than $L_B^*$, while the small lenders use the threshold*

$$L_S^{*'} = L_B^{*'} + \epsilon - \frac{2\epsilon}{1 - \lambda} \cdot m(K, R, f, d_H, \lambda).$$

***Proof.*** *See Appendix.* ■

Figure 7 shows the total proportion of investors withdrawing ex post for a given $L_0$ under the two equilibria. The feedback effect causes both the large and the small lenders to run at lower thresholds.

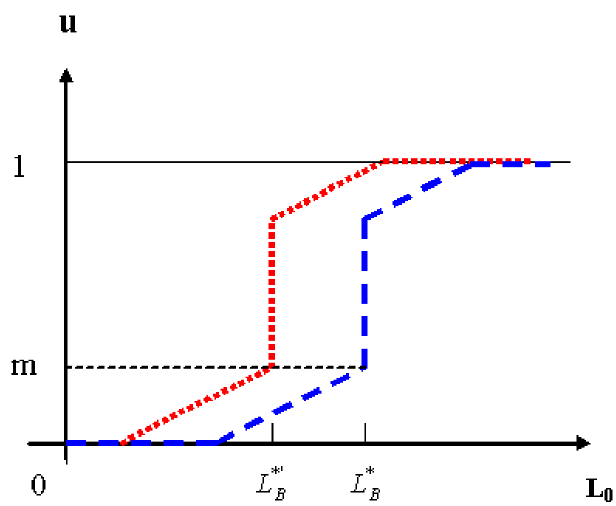


**Figure 7**: Proportion of lenders withdrawing in the two equilibria

The intuition behind Proposition 3 is as follows. When the large lender FI-B is hit by a liquidity shock, it might decide to withdraw funding to FI-H. Due to imperfect information, some small investors believe that FI-B's leverage is worse than it actually is, and therefore withdraw their funding even when sometimes FI-B does not. As FI-H must sell to satisfy those early small investors, the liquidity in market B is reduced because of the correlation between the two markets. The original shock to FI-B is therefore exacerbated, reinforcing the case for FI-B to call the loans, in turn causing more small investors to withdraw, and so on in a downward spiral. The equilibrium in the last subsection only incorporates the original shock while the current equilibrium takes into account the feedback.

The amplification mechanism in this subsection generates a liquidity spiral, in which market liquidity impacts creditors' funding availability. The lower the market liquidity, the less the creditors are able to lend (*the creditor channel*). This spiral can be contrasted with the 'margin spiral' in Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2002) where the impact of market liquidity is on the borrower's borrowing constraint, i.e., the lower the market liquidity, the lower the borrowing capacity (*the borrower channel*). Figure 8 illustrates the two spirals. The coordination problem (among the various lenders) and the liquidity spiral working through the creditor channel interact with each other.

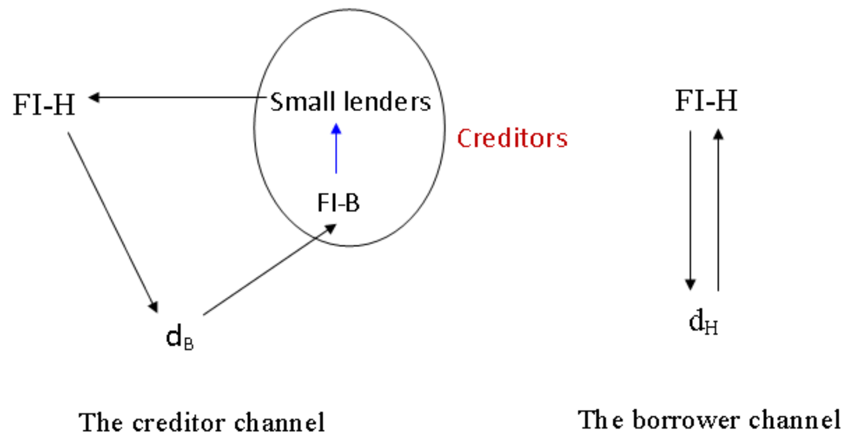


**Figure 8**: Liquidity spiral

### 2.3 Implications of the model

In this section we conduct comparative static analysis and welfare analysis.

#### 2.3.1 Comparative static analysis: how vulnerable is FI-H?

We wish to understand the magnitude and the determinants of the liquidity problems for a financial institution.

Our focus is on the borrower institution, FI-H. How likely is it to suffer from funding withdrawals by its creditors? Morris and Shin (2008) argue that two factors determine the probability of a (creditor) run. The first factor is the threshold for coordination to not run, and the second factor is the cost of miscoordination. In our model, we show that these two factors actually correspond to the two sides of the balance sheet of the borrower institution.

Looking at the first factor, which in our model is characterized by $d_B$, we can see that if $d_B$ is high, for a given $L_0$, small lenders anticipate a higher probability that the large lender FI-B will run. So it is hard for small lenders to coordinate to not run. That is,

$$\frac{\partial L_S^*(d_H, d_B)}{\partial d_B} < 0.$$

The second factor in our model is characterized by $d_H$. If $d_H$ is high, sales by FI-H will greatly depress the value of the asset. A lender that decides not to run has a high probability of getting nothing. That is, the cost of miscoordination is high. We show in the appendix that

$$\frac{\partial L_S^*(d_H, d_B)}{\partial d_H} < 0.$$

If adverse shocks occur simultaneously (i.e., $d_H$ and $d_B$ are high at the same time), a financial institution like FI-H can suffer a run that is triggered on both the asset and the liability side. When this happens, FI-H becomes very vulnerable.

In particular, if markets are dependent and market liquidity is correlated, the two channels feed into each other, creating a downward spiral. Formally, we prove in the appendix that

$$\frac{\partial^2 L_B^{*'}}{\partial d_H \partial d_B} < 0 \quad \text{and} \quad \frac{\partial^2 L_S^{*'}}{\partial d_H \partial d_B} < 0,$$

where $L_B^{*'}$ and $L_S^{*'}$ are given in Proposition 3.

### 2.3.2 Welfare analysis

Our paper emphasizes the amplification mechanisms contributing to liquidity squeezes in financial institutions. In this subsection, we discuss the welfare consequences of these amplification effects, namely the extent to which these effects are suboptimal relative to the constrained second-best equilibrium.

We define the constrained second-best equilibrium first. In our model, the small lenders do not face any constraints. The only constraint is the large lender's risk management requirement. So the constrained second-best equilibrium is the equilibrium where the large lender calls when its risk management constraint is binding and the small lenders do not call. As shown before, the large lender needs to withdraw to satisfy its risk management requirement when the liquidity shock to $d_B$ is large enough such that $d_B \geq d_B^*$, where $d_B^*$ solves $L_0 = L_B^*(d_B^*)$. Therefore, in the constrained second-best equilibrium, the aggregate proportion of lenders calling is

$$u^{SB}(L_0, d_B) = \begin{cases} 0 & \text{when } d_B < d_B^*(L_0) \\ \lambda & \text{when } d_B \geq d_B^*(L_0) \end{cases} .$$

The two amplification mechanisms in our model correspond to the two equilibria. We compare these two equilibria with the constrained second-best equilibrium in terms of welfare.

In the first equilibrium, the aggregate proportion of lenders calling, denoted by $u^I$, is $u^I(L_0, d_H, d_B) = u$, where $u$ is given by (6), while in the second equilibrium, the aggregate proportion of lenders calling, denoted by $u^{II}$, is $u^{II}(L_0, d_H, d_B) = u$, where $u$ is given by (6) but $L_B^*$ is replaced by $L_B^{*'}$ and $L_S^*$ is replaced by $L_S^{*'}$.

Clearly, we have $u^{SB} \leq u^I \leq u^{II}$, with strict inequalities holding for some $L_0, d_H, d_B$. Figure 9 shows $u^{SB}$, $u^I$ and $u^{II}$. Each level of amplification leads to more lenders withdrawing for a given $L_0$, $d_H$, and $d_B$.

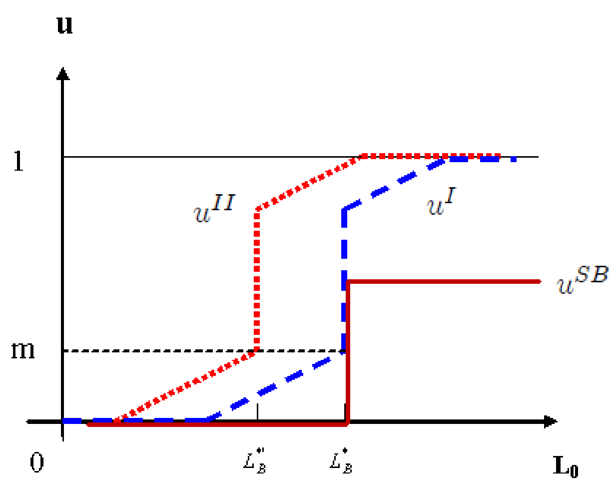


**Figure 9**: Proportion of lenders withdrawing in the three equilibria

In our model, welfare is calculated based on the total realized value of FI-H's asset (at $T_1$ and $T_2$). Specifically, if the aggregate proportion of lenders calling at $T_1$ is $u$, FI-H needs to liquidate $s$ proportion of its asset, where $u \cdot K = \pi(s)$. Therefore, the welfare, measured by the total expected liquidation value of the asset, is $W = s \cdot (f - d_H \cdot s) + (1 - s) \cdot f$, where the first (respectively second) term is the liquidation value at $T_1$ (respectively $T_2$).

We denote the welfare in the three equilibria as $W^{SB}$, $W^I$ and $W^{II}$. Because $\frac{\partial W}{\partial s} < 0$, it is easy to show that $W^{SB} \geq W^I \geq W^{II}$, with strict inequalities holding for some $L_0, d_H, d_B$.

**Proposition 4** *The welfare under the three equilibria has the ranking $W^{SB} \geq W^{I} \geq W^{II}$, with strict inequalities holding for some $L_0, d_H, d_B$. That is, the equilibria under market frictions are less efficient than the constrained second-best equilibrium while the equilibrium with the liquidity spiral is less efficient than the equilibrium without it.*

The intuition behind Proposition 4 is easy to understand. In the constrained second-best equilibrium, only the large lender calls when its risk management constraint is binding. For each level of amplification, more lenders close their positions preemptively. More sales by FI-H at a discount at $T_1$ worsen welfare.

It is worth noting that the presence of noise traders in our model does not affect the welfare analysis. In fact, noise traders' selling happens at $T_{1+}$, while our welfare analysis surrounds the selling by FI-H at $T_1$ which is triggered by the *precautionary* actions of FI-B. In other words, our welfare analysis compares the asset sales at $T_1$ under different scenarios of equilibrium *conditional on* the noise trading distribution $\widetilde{s}$ at $T_{1+}$.

# 3 Model Extensions

In this section, we consider several extensions of the main model. The extensions serve two purposes. First, we show that the basic results of our main model are robust to these extensions. Second, the extensions offer additional economic insights.

## 3.1 Debt seniority

In the main model, we have assumed that all debt is of the same seniority. In this subsection, we relax this assumption and consider the debt from FI-B to FI-H to be senior.[26] Let us consider the position of the *marginal* small lender who receives the signal $L_S^*$. The distribution of $L_0$ in his eyes is uniform $[L_S^* - \epsilon,\ L_S^* + \epsilon]$. The total proportion of small lenders withdrawing in his eyes is uniform $[0,\ 1 - \lambda]$. This marginal small lender also needs to conjecture whether the large lender will withdraw at $T_1$. From its perspective, the probability that the large lender will not withdraw is

$$\frac{L_B^* - L_S^* + \epsilon}{2\epsilon}. \tag{7}$$

[26] Dealer funding to hedge funds is typically in the form of repo or derivative contracts. Both repos and derivatives are exempt from the automatic stay, making them effectively super senior to all other claims.

In the case where FI-B withdraws, seniority implies that the marginal small lender's net expected payoff from holding (versus calling) is

$$\frac{1}{1-\lambda}\int_{\lambda}^{1} \triangle\, w(u)du.$$

In the case where FI-B does not withdraw, the marginal small lender's net expected payoff from holding (versus calling) is

$$\frac{1}{1-\lambda}\int_{0}^{1-\lambda} \triangle\, w(u)du.$$

In equilibrium, the marginal small lender is indifferent to holding versus calling, which means

$$\left(1-\frac{L_B^*-L_S^*+\epsilon}{2\epsilon}\right)\frac{1}{1-\lambda}\int_{\lambda}^{1} \triangle\, w(u)du+\frac{L_B^*-L_S^*+\epsilon}{2\epsilon}\frac{1}{1-\lambda}\int_{0}^{1-\lambda} \triangle\, w(u)du=0. \tag{8}$$

Hence, we find the equilibrium.

**Proposition 5** *Suppose that the lending by the large lender FI-B is senior to the lending by small lenders. In equilibrium, the large lender's threshold is $L_B^*$, as given in Lemma 1, while the small lenders' threshold is*

$$L_S^*(d_H,d_B)=L_B^*(d_B)+\epsilon-\frac{2\epsilon}{1-\lambda}\cdot m(K,R,f,d_H,\lambda),$$

*where* $m=(1-\lambda)\dfrac{-\int_{\lambda}^{1}\triangle w(u)du}{-\int_{\lambda}^{1}\triangle w(u)du+\int_{0}^{1-\lambda}\triangle w(u)du}$.

***Proof.*** *See Appendix.* ■

By proposition 5, the result in Corollary 1 does not change; namely in equilibrium there is a proportion, $m$, of small lenders that run before the larger lender does (although $m$ is different from that in Corollary 1). Hence, the results in Propositions 3 and 4 do not change.

### 3.2 The small investors as equity investors

So far we have assumed that the small lenders in our model are debtholders. Now we will explicitly model them as limited partners in a hedge fund — equity investors. Liu and Mello (2011) model runs on hedge funds with no debt (and only equity). In that article, we show that equity partners in funds who are able to redeem at short notice have a payoff structure resembling that of short-term debtholders, and hence a first-mover advantage causing a run exists.

Now we consider a hedge fund (FI-H) with internal equity, external equity (small investors), and debt (FI-B's lending), as depicted in Figure 10. The lender, FI-B, can run because of margin calls, and the external equity investors can also run because of redemption. At $T_0$, the asset side of FI-H includes one unit of the asset with its market price being 1. To finance the asset, FI-H borrows $K$ (face value) from FI-B, and the remaining financing comes from a continuum of equityholders with unit mass, each contributing $1 - K$. Among the equityholders, the $\lambda$ proportion consists of internal equityholders and the $1 - \lambda$ proportion external equityholders. Other setups are the same as those of the main model.[27]

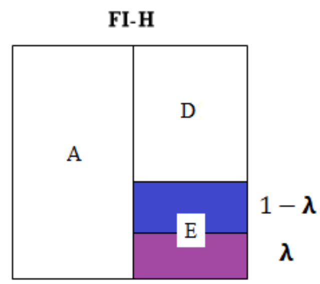


**Figure 10**: Small investors as equity investors in FI-H

The key difference between this extension and the main model lies in the payoff structure, which is no longer that in Lemma 2, that is, Figure 2 will be different. Denote by $u$ the number of small investors that redeem at $T_1$, where $0 \leq u \leq 1 - \lambda$. Conditional on the broker FI-B calling its loan at $T_1$, the payoff for an individual small investor choosing to redeem versus not to redeem is, respectively, given by

$$\bar{w}^C(u) = \begin{cases} 1 - K & \text{if } \; 0 \leq u \leq \frac{f - d_H - K}{1-K} \\ \frac{f - d_H - K}{u} & \text{if } \; \frac{f-d_H-K}{1-K} < u \leq 1 - \lambda \end{cases} \tag{9}$$

and

$$\bar{w}^H(u) = \begin{cases} \frac{(2d_H - f) + \sqrt{f^2 - 4d_H[K + u(1-K)]}}{2d_H(1-u)} f & \text{if } \; 0 \leq u \leq \frac{f-d_H-K}{1-K} \\ 0 & \text{if } \; \frac{f-d_H-K}{1-K} < u \leq 1 - \lambda \end{cases}. \tag{10}$$

In contrast, conditional on FI-B not calling its loan at $T_1$, the payoff for an individual small investor choosing to redeem versus not to redeem is, respectively, given by

$$\hat{w}^C(u) = 1 - K \qquad \text{for } 0 \leq u \leq 1 - \lambda \tag{11}$$

[27] For simplicity, we assume that the asset of FI-A has payoff $f$ at $T_2$, without uncertainty.

and

$$\hat{w}^{H}(u)=\frac{\max\left[\frac{(2d_H-f)+\sqrt{f^2-4d_Hu(1-K)}}{2d_H}f\ \ -KR,0\right]}{1-u}\qquad\text{for}\ \ 0\leq u\leq 1-\lambda. \tag{12}$$

To see (9)-(12), notice first that at $T_1$ the hedge fund's mark-to-market net asset value (NAV) is $1-K$.[28] If a small investor redeems, his claim is $1-K$ at $T_1$. Also, the loan from FI-B is senior. We discuss two scenarios in order. In the first scenario FI-B calls its loan at $T_1$. Conditional on a total number, $u$, of small investors redeeming at $T_1$, FI-H needs to sell $s$ units of its asset, where $\pi(s)=K+u(1-K)$ or $s=\frac{f-\sqrt{f^2-4d_H[K+u(1-K)]}}{2d_H}$. If $u$ is small enough such that $\pi(1)\geq K+u(1-K)$ or $u\leq\frac{f-d_H-K}{1-K}$, FI-H does not fail at $T_1$, in which case a small investor that redeems has payoff $\bar{w}^{C}(u)=1-K$ and a small investor that does not redeem has payoff $\bar{w}^{H}(u)=\frac{(1-s)f}{1-u}$. If $u$ is high enough such that $\pi(1)<K+u(1-K)$ or $u>\frac{f-d_H-K}{1-K}$, FI-H fails at $T_1$, in which case a small investor that redeems has payoff $w^{C}(u)=\frac{f-d_H-K}{u}$ and a small investor that does not redeem has payoff $w^{H}(u)=0$.

In the second scenario FI-B does not call its loan at $T_1$. For simplicity, we assume that $\pi(1)\geq(1-\lambda)(1-K)$ or $f-d_H>(1-\lambda)(1-K)$, which means that even if all small investors redeem at $T_1$, FI-H can still survive at $T_1$. Conditional on a total number, $u$, of small investors redeeming at $T_1$, FI-H needs to sell $s$ units of its asset, where $\pi(s)=u(1-K)$ or $s=\frac{f-\sqrt{f^2-4d_Hu(1-K)}}{2d_H}$. Hence, an individual small investor that redeems has payoff $\hat{w}^{C}(u)=1-K$, which is (11); an individual small investor that does not has payoff $\hat{w}^{H}(u)=\frac{\max[(1-s)f\ \ -KR,0]}{1-u}$, which is (12), by noting that FI-H needs to first repay FI-B the debt claim $KR$ and then distribute the fund's payoff to its staying equityholders at $T_2$.

Define $\triangle\bar{w}(u)\equiv\bar{w}^{H}(u)-\bar{w}^{C}(u)$ and $\triangle\hat{w}(u)\equiv\hat{w}^{H}(u)-\hat{w}^{C}(u)$. As the information structure is still the same as in the main model, from the perspective of the marginal small investor, the probability that FI-B will not withdraw is still given by (7). In equilibrium, the marginal small investor is indifferent to redeeming versus not, which implies

$$\left(1-\frac{L_B^*-L_S^*+\epsilon}{2\epsilon}\right)\frac{1}{1-\lambda}\int_0^{1-\lambda}\triangle\bar{w}(u)du+\frac{L_B^*-L_S^*+\epsilon}{2\epsilon}\frac{1}{1-\lambda}\int_0^{1-\lambda}\triangle\hat{w}(u)du=0.$$

**Proposition 6** *Suppose that the small investors are equity investors that can redeem at* $T_1$. *In*

[28] When investors give the fund notice of their redemptions, the hedge fund's mark-to-market asset value is 1. As long as it starts to liquidate its asset, the realized liquidation value is given by the downward-sloping sale price.

*equilibrium, FI-B's threshold is $L_B^*$, as given in Lemma 1, while the small investors' threshold is*

$$L_S^*(d_H, d_B) = L_B^*(d_B) + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m(K, R, f, d_H, \lambda),$$

*where* $m = (1-\lambda)\frac{-\int_0^{1-\lambda} \triangle\bar{w}(u)du}{-\int_0^{1-\lambda} \triangle\bar{w}(u)du + \int_0^{1-\lambda} \triangle\hat{w}(u)du}$.

***Proof.*** *See Appendix.* ■

Proposition 6 means that, in equilibrium, there is a proportion $m$ of small investors that run before the large lender does (where $m$ is different from the one in Corollary 1). Hence, the results in Propositions 3 and 4 do not change.

In order to mitigate the potential run by investors, a hedge fund can incorporate gates and redemption fees in the contract with their client investors. We will now analyze the impact of these measures on runs. First, suppose that there exist fund-level gates that limit the total redemptions at $T_1$ up to $\kappa$ proportion of the overall net asset value of the hedge fund, where $\kappa \leq 1-\lambda$. Then, the equilibrium is given by

$$\left(1 - \frac{L_B^* - L_S^* + \epsilon}{2\epsilon}\right) \frac{1}{1-\lambda} \int_0^{\kappa} \triangle\bar{w}(u)du + \frac{L_B^* - L_S^* + \epsilon}{2\epsilon} \frac{1}{1-\lambda} \int_0^{\kappa} \triangle\hat{w}(u)du = 0.$$

**Corollary 2** ***(Gate Provisions)*** *Suppose the hedge fund contract has fund-level gates where the maximum allowable redemption is $\kappa$ proportion of the overall net asset value. Then, $m(K, R, f, d_H, \lambda)$ in Proposition 6 becomes*

$$m = (1-\lambda)\frac{-\int_0^{\kappa} \triangle\bar{w}(u)du}{-\int_0^{\kappa} \triangle\bar{w}(u)du + \int_0^{\kappa} \triangle\hat{w}(u)du}.$$

*We have comparative statics that $L_S^*$ is decreasing in $\kappa$ (for some region of $\kappa$).*

***Proof.*** *See Appendix.* ■

Corollary 2 says that when hedge funds incorporate gates, small investors have less incentive to run. The intuition is easy to understand. When there is an upper limit on redemption, the risk of miscoordination among small investors in rollover becomes lower (that is, an individual small investor becomes less worried that other small investors may run on the fund). So in equilibrium, every small investor becomes more willing to stay with the fund and sets a higher $L_S^*$.

Second, suppose that the hedge fund charges a small investor a redemption fee $\delta$ for redemptions at $T_1$.[29] Then, the equilibrium is given by

$$\left(1-\frac{L_B^*-L_S^*+\epsilon}{2\epsilon}\right)\frac{1}{1-\lambda}\int_0^{1-\lambda}\triangle\bar{w}(u)du+\frac{L_B^*-L_S^*+\epsilon}{2\epsilon}\frac{1}{1-\lambda}\int_0^{1-\lambda}\triangle\hat{w}(u)du=-\delta.$$

**Corollary 3** ***(Redemption Fees)*** *Suppose that the hedge fund imposes a redemption fees $\delta$. Then, $m(K,R,f,d_H,\lambda)$ in Proposition 6 becomes*

$$m=(1-\lambda)\frac{-\int_0^{1-\lambda}\triangle\bar{w}(u)du-\delta}{-\int_0^{1-\lambda}\triangle\bar{w}(u)du+\int_0^{1-\lambda}\triangle\hat{w}(u)du}.$$

*We have comparative statics that $L_S^*$ is increasing in $\delta$.*

***Proof.*** *See Appendix.* ■

Corollary 3 clearly shows that the existence of redemption fees discourages small investors from running, leading them to set a higher running threshold $L_S^*$.

## 3.3 Welfare implications of provisions and regulations

We can extend the welfare analysis to consider the effect of minimum haircuts in the dealer to the hedge fund market, the role of the exemption from the automatic stay of the dealer funding (in the case of repos and derivative funding), as well as capital and liquidity requirements for the dealers. In the context of our model, the effect of all these provisions and regulations is essentially embodied in the way the changes in the balance sheet position of FI-B impact its decision to lend to FI-H. Conveniently, we can model the effect by studying the comparative statics on parameters $\overline{L}$ and $\alpha$. A lower $\overline{L}$ means tighter capital and liquidity requirements. A higher $\alpha$ means that even though market liquidity $d_B$ has changed, the lending policy of FI-B to FI-H is less affected; hence, a lower $\alpha$ can be mapped into the provisions of higher minimum haircuts and of greater exemption from the automatic stay of the dealer funding. It is easy to obtain the following comparative statics:

$$\frac{\partial L_B^*}{\partial\alpha}>0 \text{ and } \frac{\partial L_B^*}{\partial\overline{L}}>0.$$

In Figure 9, a higher $\alpha$ or $\overline{L}$ shifts the curve to the right, corresponding to a welfare gain.

[29] For simplicity, we assume that the fees go to the hedge fund's unlimited partners — internal equityholders.

## 3.4 Rationale behind the Value-at-Risk (VaR) constraint

In the main model, we have taken the VaR risk management as exogenous. VaR can be rationalized in a broader model that incorporates both the downside and the upside of the VaR constraint. The basic intuition is as follows. Financial firms suffer from moral hazard problems with insider managers (from the CEO all the way down to loan and investment officers), such as excessive risk taking, leniency in screening on loans and investment, and shirking in monitoring on investment and lending. VaR risk management is an effective internal control device that helps to mitigate the moral hazard problems.

In the context of our model, we can extend the main model in a simple way to rationalize the VaR constraint. Without the VaR constraint in place at the interim date $T_1$, the managers' compensation could only be determined based on their performance at the final date $T_2$; in this case, the managers would have incentives to behave opportunistically and conduct (off-equilibrium) *excessive risk-taking* in investment ex ante at $T_0$ leading to a *negative* (expected) NPV in the spirit of risk shifting in Jensen and Meckling (1976). By contrast, with the threat of the VaR constraint, the managers are disciplined not to take excessive risk ex ante, otherwise the VaR constraint will be violated at the interim date $T_1$ and the managers will be punished/fired. This is the upside of the VaR constraint. Of course, when an aggregate shock hits (i.e., an adverse shock on $d_B$, perhaps with a small probability ex ante), the VaR constraint has negative consequences, which is the focus of study of our main model.

## 3.5 Optimal creditor structure of FI-H

In this subsection, we extend the main model to analyze the optimal creditor structure for a financial institution, by making $\lambda$ endogenous. The analysis incorporates both the upside and the downside of wholesale financing.

We slightly modify the setup of the main model to have a more abstract creditor-run game. We assume that FI-H's long-term asset realizes one of two payoffs at $T_2$, $\{X, 0\}$, where $X > 0$. Ex ante, the probability that $X$ is realized is $\theta$. The term $\theta$ represents the fundamentals of the asset held by FI-H, and $\theta$ has an (improper) uniform prior over the real line. The liquidation value of the long-term asset at $T_1$ is a constant $L$, where $L < K$. Recall that the face value of FI-H's debt at $T_1$ is $K$, with notional value at $T_2$ of $KR$. Therefore, if a proportion greater than $\frac{L}{K}$ of the creditors call the loans at $T_1$, the liquidation value of the asset will not be sufficient to satisfy the creditors' claims, and consequently FI-H will fail. Alternatively, one might think of $L$ as the available collateral from FI-H's asset. This means that FI-H is able to raise an amount of cash at $T_1$ no larger than $L$. If the demand of FI-H's creditors for cash exceeds $L$ at $T_1$, FI-H will fail in a classic bank run fashion.

Following the work of Rochet and Vives (2004) and Morris and Shin (2009), Table 1 shows the simplified creditor-run payoff structure.

| | Calling proportion no greater than $\frac{L}{K}$ (FI-H survives) | Calling proportion greater than $\frac{L}{K}$ (FI-H fails) |
|---|---|---|
| Hold | $KR$ (at $T_2$) | $0$ (at $T_2$) |
| Call | $K$ (at $T_1$) | $K$ (at $T_1$) |

**Table 1**: Creditor-run payoff structure of the extended model

To make the problem interesting, we focus on the case where the proportion of the loan from the large lender is not too large. Concretely, we consider $\lambda \leq \frac{L}{K} \leq 1 - \lambda$ (that is, $\lambda \leq \min\{\frac{L}{K}, 1 - \frac{L}{K}\}$), where $\lambda$ represents the proportion of FI-B's loan in FI-H's total debt. The first inequality means that a run solely by the large lender is not sufficient to cause FI-H to fail; the second inequality means that if all the small lenders run, FI-H may fail even if the larger lender stays put.

We consider that FI-H's creditors face *fundamental risk* in addition to the *coordination risk* among themselves. Fundamental risk was not necessary in analyzing the two amplification mechanisms in the main model, but is now helpful for studying in a simple way the optimal $\lambda$. Specifically, at $T_1$, both the large lender FI-B and the small lenders receive imperfect information (signals) about $\theta$. The large lender observes the realization of the random variable $\theta^B = \theta + \delta\eta$, where $\delta > 0$ is a constant and $\eta$ is a random variable with mean zero and smooth symmetric density $h(\cdot)$, and cumulative distribution function $H(\cdot)$. Similarly, the small lender $i$'s signal about $\theta$ is $\theta^i = \theta + \sigma\epsilon^i$, where $\sigma > 0$ is constant, and the individual specific noise $\epsilon^i$ is distributed according to the smooth asymmetric density $g(\cdot)$ (denoting its c.d.f. by $G(\cdot)$). $\epsilon^i$ is i.i.d. across small lenders, and each is independent of $\eta$.[30]

[30] The probability $\theta$ lies in the interval $[0, 1]$ while the signal's range is $[-\infty, +\infty]$. However, this is for technical

FI-B may be constrained at $T_1$, in which case it has to call its loans from FI-H. At $T_1$, all (small) lenders receive a common signal $q$ ($\in [0,1]$) — the probability of FI-B being constrained; FI-B knows perfectly whether it itself is constrained or not. The signal $q$ is equivalent to public information in global games. Ex ante, at $T_0$, $q$ has probability density $z(q)$.

We conduct the analysis by backward induction, from $T_1$ to $T_0$.

### 3.5.1 Equilibrium at $T_1$

We are interested in the equilibrium where every lender uses a threshold strategy. The strategy of the small lenders is given by $(\theta^i, q) \longmapsto \begin{cases} Call \;\; \theta^i \leq x^*(q) \\ Hold \;\; \theta^i > x^*(q) \end{cases}$, where $x^*(q)$ is the threshold. The strategy of the large lender is given by $\theta^B \longmapsto \begin{cases} Call \;\; \theta^B \leq y^*(q) \\ Hold \;\; \theta^B > y^*(q) \end{cases}$, where $y^*(q)$ is the threshold.[31]

We consider first the small lenders' strategy. By symmetric equilibrium (among small lenders), conditional on all other small lenders using the threshold strategy with the threshold being $x^*$ and the large lender using the threshold $y^*$, an individual small lender's optimal threshold should be $x^*$ as well. That is,

$$\begin{aligned} \int_{-\infty}^{+\infty} \{ I(\Phi(\theta; x^*)(1-\lambda) \;\; &\leq \;\; \frac{L}{K}) \cdot (KR\cdot\theta) \cdot \left[(1-q)\Pr(\theta^B > y^*|\theta)\right] + \\ I(\Phi(\theta; x^*)(1-\lambda) + \lambda \;\; &\leq \;\; \frac{L}{K}) \cdot (KR\cdot\theta) \cdot \left[q + (1-q)\Pr(\theta^B \leq y^*|\theta)\right] \} g(\theta|x^*) d\theta = K, \end{aligned} \tag{13}$$

where $I$ is a logical function defined as $I(t) = \begin{cases} 1 & \text{if } t \text{ is true} \\ 0 & \text{otherwise} \end{cases}$, $\Phi(\theta; x^*) = G(\frac{x^*-\theta}{\sigma})$, measuring the fraction of small lenders that withdraw for a given realization of $\theta$ when all small lenders use the threshold $x^*$, and $g(\theta|x^*) = \frac{dG(\frac{\theta - x^*}{\sigma})}{d\theta}$ is the posterior density function.

Equation (13) expresses the decision of the marginal small lender whose signal is just equal to $x^*$; the LHS is its conditional expected payoff for not withdrawing while the RHS is the payoff

convenience. In fact, later when we consider the limiting case of $\delta \to 0$ and $\sigma \to 0$, the probability of signals falling outside $[0,1]$ is negligible. In particular, as in Goldstein and Pauzner (2005), we can use a more complicated setup where there is a one-to-one mapping $\theta \in [0,1] \to [-\infty, +\infty]$, and agents receive signals about the value of the mapping rather than directly about $\theta$.

[31] The large lender's threshold $y^*$ is also a function of $q$. The large lender knows that the small lenders know $q$, the higher order beliefs.

for withdrawing. For a given realization of $\theta$, there is no uncertainty on the aggregate fraction of the small lenders withdrawing, that is, $\Phi(\theta; x^*)$ is determined. The large lender, however, may or may not withdraw, and that depends on whether it is constrained, as well as on the strength of its signal; for a given realization of $\theta$, the probability of the large lender not withdrawing is $(1-q)\Pr(\theta^B > y^*|\theta)$, and the probability of withdrawing is $q + (1-q)\Pr(\theta^B \leq y^*|\theta)$. Conditional on FI-B not withdrawing, FI-H survives at $T_1$ if $\Phi(\theta; x^*)(1-\lambda) \leq \frac{L}{K}$; and conditional on FI-B withdrawing, FI-H survives when $\Phi(\theta; x^*)(1-\lambda) + \lambda \leq \frac{L}{K}$. If FI-H survives at $T_1$, the lender's expected payoff at $T_2$ is $KR \cdot \theta$ for a given realization of $\theta$. Because the lender does not know $\theta$ and only receives a signal $x^*$, its conditional expected payoff is the LHS of equation (13).

Consider next FI-B's strategy. If FI-B is constrained at $T_1$, it withdraws the loans to FI-H. If FI-B is not constrained at $T_1$, its optimal threshold $y^*$ satisfies the condition

$$\int_{-\infty}^{+\infty} \left[ I(\Phi(\theta; x^*)(1-\lambda) \leq \frac{L}{K}) \cdot (KR \cdot \theta) \right] h(\theta|y^*) d\theta = K, \tag{14}$$

where $h(\theta|y^*) = \frac{dH(\frac{\theta - y^*}{\delta})}{d\theta}$ is the posterior density function. The LHS of equation (14) represents FI-B's expected payoff if it does not withdraw, while the RHS is FI-B's payoff if it withdraws.[32] Note that on the LHS, since FI-B does not withdraw and only the small lenders might withdraw, FI-B's expected payoff is $I(\Phi(\theta; x^*)(1-\lambda) \leq \frac{L}{K}) \cdot (KR \cdot \theta)$ for a given realization of $\theta$.

The equilibrium of the creditor-run game is characterized by equations (13) and (14). Proposition 7 follows.

**Lemma 3** *There is a unique threshold equilibrium for the creditor-run game that solves the system of equations (13)-(14).*

***Proof.*** *See Appendix.* ■

We now proceed to characterize the equilibrium. As in Corsetti et al. (2004), we characterize the equilibrium by focusing on the limit case where $\delta \to 0$, $\sigma \to 0$, and $\frac{\delta}{\sigma} \to c$. In other words, both the large lender and the small lenders have precise information, but the noise of the large lender's signal relative to that of the small lenders' signals tends to $c$, with $0 \leq c \leq +\infty$. The limit case is used for tractability. We can show that in the limit, FI-H fails at $T_1$ if and only if $\theta \leq \theta^*$, where $\theta^*$ denotes the failure threshold of FI-H (see the appendix).

[32] Both sides of equation (14) should be multiplied by $\lambda$.

We conduct comparative statics and provide an answer to the following question: How does the failure threshold $\theta^*$ behave as $\lambda$ and $q$ change? We express $\theta^*$ as a function of $\lambda$ with parameter $q$, written as $\theta^*(\lambda; q)$), which has the following property.

**Lemma 4** *For a given $q$, $\theta^*(\lambda; q)$ is increasing in $\lambda$ when $q$ is sufficiently high, and decreasing in $\lambda$ when $q$ is sufficiently low. For a given $\lambda$, an increase in $q$ leads to an increase in $\theta^*$ (point-wisely) in an upward spiral ($\frac{\partial x^*}{\partial q} > 0$, $\frac{\partial y^*}{\partial x^*} \geq 0$, $\frac{\partial x^*}{\partial y^*} > 0$).*

***Proof.*** *See Appendix.* ■

Lemma 4 says that the presence of a large lender can act as either a stabilizing or a destabilizing force. When the large lender is more likely to be constrained (i.e., a higher $q$), the larger the size of $\lambda$, the more fragile FI-H is. On the opposite, when the large lender is less likely to be constrained (i.e., a lower $q$), the larger the size of $\lambda$, the less fragile FI-H is. The two polar cases of $q = 0$ and $q = 1$ help illustrate the intuition. With $q = 0$, the large lender knows perfectly whether it itself will withdraw and thus faces the risk of coordination only with the small lenders. By contrast, each individual small lender (with a negligible mass) faces the risk of coordination with other small lenders, as well as with the large lender. Therefore, the presence of the large lender reduces the overall difficulty of coordination among the lenders. With $q = 1$, the large lender runs, and therefore it is harder for the remaining lenders (i.e., the small lenders) to coordinate to not run as $\lambda$ increases. They thus run sooner (at a higher threshold), making FI-H's failure more likely.

Figure 11 reports a numerical example of $\theta^*(\lambda; q)$ for different values of $q$, where the parameters are $K = 1$, $L = 0.52$, $R = 2.5$, $\delta = \sigma = 0.0001$, and $\min\{\frac{L}{K}, 1 - \frac{L}{K}\} = 0.48$. $H$ and $G$ are standard normal.

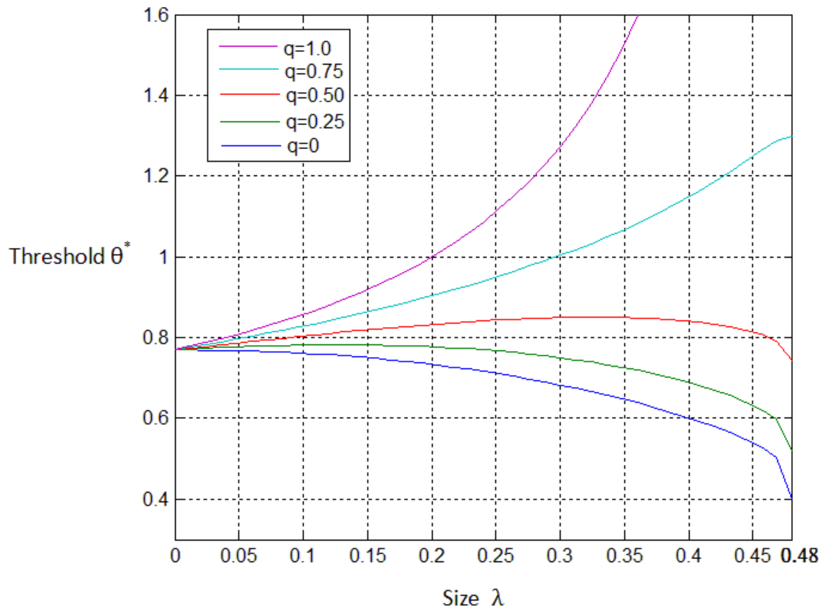


**Figure 11**: Function $\theta^*(\lambda; q)$

### 3.5.2 Equilibrium at $T_0$

We are now able to address the main question in this extended model: does an optimal creditor structure for FI-H exist (i.e., an optimal size of $\lambda$)? Ex ante, at $T_0$, $q$ has the probability distribution $z(q)$. FI-H aims to minimize the expected value of the failure threshold $\theta^*$ at $T_1$. Therefore, FI-H's optimization problem at $T_0$ is

$$\min_{\lambda} \int_0^1 \theta^*(\lambda; q) z(q) dq \tag{15}$$

In general, the optimization problem in (15) has a unique interior solution.

**Proposition 7** *Under some probability distribution $z(q)$, there is a unique optimal $\lambda \in (0, 1)$. That is, FI-H has a unique optimal creditor structure.*

***Proof.*** *See Appendix.* ■

The intuition behind Proposition 7 is the following. The presence of a large lender can be good or bad, depending on the state of $q$ at $T_1$. When $q$ is high, the presence of the large lender increases the fragility of FI-H. Conversely, when $q$ is low, the presence of the large lender helps mitigate the coordination problem among the lenders, reducing the fragility of FI-H. Ex ante, given a probability distribution of $q$, FI-H chooses a unique optimal size of $\lambda$ at $T_0$.

Continuing the numerical example above, we choose a simple discrete distribution for $z(q)$ where $z(q) = \begin{cases} 0.9 & \text{for } q = 0 \\ 0.1 & \text{for } q = 1 \end{cases}$. In this case, we find that the optimal $\lambda$ is $\lambda^* = 0.36$, which is between 0 and $\min\{\frac{L}{K}, 1 - \frac{L}{K}\} = 0.48$.

### 3.5.3 Endogenous $q$

Suppose that the probability $q$ is not a constant at $T_1$ but a function $q(\Phi;\omega)$, where $\Phi$ is the aggregate withdrawals by FI-H's small lenders (i.e., $\Phi(\theta; x^*) = G(\frac{x^*-\theta}{\sigma})$) and $\omega$ is a parameter describing market conditions or the degree of systemic risk at $T_1$, with $\frac{\partial q(\Phi;\omega)}{\partial \Phi} > 0$ and $\frac{\partial q(\Phi;\omega)}{\partial \omega} > 0$. The economic interpretation of $q(\Phi;\omega)$ is the following. The withdrawals by its creditors impact FI-H's financial health, in turn affecting the liquidity of the market where FI-B operates and, therefore, FI-B's funding availability. That is, the probability that FI-B is constrained is partly determined by FI-H's status such as $\Phi$. Essentially, $q(\Phi;\omega)$ is a reduced-form result of the micro-foundation analysis in Section 2.2.2.

At $T_1$, all the small lenders in FI-H observe $\omega$ (instead of a constant $q$) and their threshold strategy becomes $(\theta^i, \omega) \longmapsto \begin{cases} Call \;\; \theta^i \le x^*(\omega) \\ Hold \; \theta^i > x^*(\omega) \end{cases}$, where $x^*(\omega)$ is the threshold. We can show that an endogenous $q$ creates additional amplifications. In fact, when we substitute $q(\Phi;\omega)$ for $q$ in equation (13), an additional feedback loop emerges: $\frac{\partial q}{\partial \Phi} > 0$, $\frac{\partial x^*}{\partial q} > 0$ and $\frac{\partial \Phi}{\partial x^*} > 0$. This feedback loop (i.e., the second amplification) and the original feedback loop (i.e., the first amplification) interact and reinforce each other, and form a compound spiraling effect. Formally, $\frac{\partial q}{\partial \omega} > 0$; $\frac{\partial x^*}{\partial q} > 0$, $\frac{\partial y^*}{\partial x^*} \ge 0$, $\frac{\partial x^*}{\partial y^*} > 0$, $\frac{\partial \Phi}{\partial x^*} > 0$, $\frac{\partial q}{\partial \Phi} > 0$. Therefore, a small shock to the realization of $\omega$ at $T_1$ can have a large impact on the equilibrium $\theta^*$ when solving the system of equations (13)-(14).

At $T_0$, FI-H knows the probability distribution of $\omega$, and chooses an optimal $\lambda$ that minimizes the expected value of the failure threshold $\theta^*$ at $T_1$.

In sum, if we incorporate the liquidity spiral into the current extended model, the optimal size of the large lender $\lambda$ can become smaller because of this additional negative side of wholesale financing (i.e., the downward liquidity spiral caused). That is, at $T_0$ FI-H relies less on the large lender's funding.

# 4 Empirical Evidence and Implications

We discuss the empirical implications of the model and the evidence on hedge funds.

## 4.1 The creditor channel

Hedge funds have often been seen as a source of risk in modern financial markets. Since the event of LTCM, regulators have endeavored to impose tough regulations on hedge funds. However, the 2007-2009 crisis has shown that it was the banking sector, in their role as creditors, that spread the financial problems to the hedge fund sector, and not the other way round. Evidence of this is mounting. Mitchell and Pulvino (2012) document the spread of liquidity shocks from the broker-dealer sector to the hedge fund sector, in turn, limiting arbitrages of hedge funds. Aragon and Strahan (2012) find that the financial problems of prime brokers interrupted the activities of hedge funds. Along this line, the *Economist* wrote in the middle of the crisis: "Regulators used to worry about the danger hedge funds might pose to their prime brokers... the risk turned out to be the other way round."[33]

In this paper we highlight one reason for bank credit to stop flowing: the precautionary liquidity hoarding of market-based lenders. Our explanation relies on the fact that lenders nowadays are often market-based leveraged institutions, and are therefore subject to market conditions.[34] Acharya and Merrouche (2013) document banks hoarding liquidity for precautionary motives in the 2007-2009 crisis. The findings of Shin (2009) support the view that the reduction of exposures by lenders arises from risk management of potential losses.

## 4.2 The liquidity-triggered run

In our model, small investors play an important role. They are long-term investors not subject to balance sheet constraints, and therefore they can be seen as true spare liquidity providers to the financial system. However, they may decide to withdraw if they become worried that other investors might withdraw, in particular the large lender. Therefore, the run by small investors in our model

[33] "Hedge Funds in Trouble: The Incredible Shrinking Funds", *Economist*, October 25-31 2008, pp. 87-88.

[34] This explanation differs from and complements the typical financial-contagion explanation (see, e.g., Allen and Gale (2000)) where there is some *realized loss* (in some other geographic locations) of traders. In our paper, market liquidity changes, causing the risk of *potential loss* (in lenders' asset base), explain a credit reversal.

is triggered by concerns on the liability side.[35] Importantly, the run by small investors in our model is not purely self-fulfilling (i.e., sunspot equilibrium) as in Diamond and Dybvig (1983). Instead, small investors have material reasons to run, as they understand that some other (large) lenders may be facing balance sheet constraints and are likely to withdraw. Putting it slightly differently, the run in our model does not arise out of the blue; rather, the run is triggered when some market-based large lenders are suspected of becoming constrained and threatening the liquidity status of the borrower.

According to the International Financial Services London (IFSL), the hedge fund industry suffered severe runs and faced unprecedented pressure from investors for redemptions in 2008. Based on their estimates, investors pulled more than $300 billion from hedge funds in the second half of 2008.[36] Interestingly, the *Economist* wrote, "The [hedge] industry's aggregate leverage has undoubtedly caused it trouble. But there does not appear to have been a systematic withdrawal of bank credit from hedge funds....A fuller explanation must include the increasingly jittery nature of hedge funds' clients". We argue that one important reason for the flight of hedge fund investors was that the primary brokers at banks were in trouble and bank lending was tightening. Investors worried that the tightening of bank credit would force hedge funds to liquidate positions at big losses. They got scared and rushed to exit.[37]

### 4.3 The feedback effect

Our model highlights a second amplification mechanism — the feedback from the hedge fund sector to the banking sector. Some evidence is available. Aragon and Strahan (2012), using the September 15, 2008 bankruptcy of Lehman Brothers as an exogenous shock, find that stocks traded by Lehman-connected hedge funds experienced greater declines in market liquidity following the

[35] Received wisdom usually attributes the trigger for the bank run on Northern Rock, a UK bank, to its mortgage assets. Shin (2009), however, provides convincing evidence showing that this was not the main reason for the run. Shin argues: "[Northern Rock] was with an apparently solid asset book, with virtually no subprime lending... Rather, its problems stemmed from its high leverage coupled with reliance on institutional investors for short term funding. When the de-leveraging in the credit markets began in August 2007, it was uniquely vulnerable to the shrinking of lender balance sheets arising from the tick-up in measured risks." According to Shin (2009), the run on Northern Rock stemmed from its large lenders, the liability side of its balance sheet, rather than from the asset side.

[36] "Hedge Funds 2009", the International Financial Services London Research (IFSL).

[37] Ang et al. (2011) find that during the recent financial crisis hedge fund leverage reduced precisely when large financial intermediates, an important source of lending for hedge funds, became more financially constrained.

bankruptcy than other stocks. They thus conclude that hedge funds are market-liquidity providers. Mitchell and Pulvino (2012) show that the crippling of hedge funds led to high mispricings in the financial markets, meaning that the markets became less informationally efficient and less liquid. Coval and Stafford (2007) and Hau and Lai (2011) also provide evidence along the same line. We argue that the reduced market liquidity in turn impacts banks' decisions. When market liquidity dries up, banks face greater uncertainty in their balance sheets, and tend to reduce exposure and hoard liquidity.

## 5 Concluding Remarks

This paper analyzes the transmission of liquidity shocks through the creditor channel and highlights two intertwining amplification mechanisms. We are interested in a positive question: what do financial institutions do and what is the chain reaction given the financial structure that financial institutions have adopted in practice? In future work, we wish to study a related normative question: what is the ex-ante optimal structure?

With respect to policy implications, we remark that in modern financial markets it is important to consider not just the risk of the assets of a financial institution, but also the risk that stems from both the degree of indebtedness and the structure of its creditors. Institutions that depend in a significant way on funds from the wholesale markets must be closely monitored, and may need to satisfy particular risk-control requirements. Financial regulators must consider whether limits on leverage combined with limits on the relative share of lending by large lenders, when these are financial investors, are the more appropriate regulations. Regulators must take a systemic view to monitor both borrowers' behavior and lenders' actions, because what happens to these lenders may drastically affect not just the borrower institution directly but also indirectly the actions of other lenders.

## Appendix

# A Proofs

**Proof of Lemma 1:** By applying $A^S + A^L_{1+} = D + E_{1+}$, we can rewrite (3) as

$$\Pr(A^L_{1+} < D\frac{\overline{L}}{\overline{L}-1} - A^S) < \alpha. \tag{A.1}$$

Denote the $\alpha$-percentile of $A^L_{1+} \sim N(v, \sigma^2 d_B^2)$ as $C$, that is, $\alpha \equiv \Phi(\frac{C-v}{\sigma d_B})$. Obviously, $C(d_B)$ is a decreasing function with respect to $d_B$, considering that $\alpha < \frac{1}{2}$. Using $d_B$, (A.1) is transformed into

$$D\frac{\overline{L}}{\overline{L}-1} - A^S < C(d_B). \tag{A.2}$$

At $T_1$, FI-B receives perfect information about the market depth, $d_B$. If $d_B$ is high, the market is illiquid and the RHS of (A.2) is low. In this case, the inequality (A.2) may be violated. Then, to comply with risk management requirements and satisfy (A.2), FI-B delevers. It does so by calling the short-term loans extended to FI-H and using the proceeds to repay the debt. To see this, suppose that FI-B calls the short-term loans in the amount of $\triangle A^S > 0$, and repays its outstanding debt. The LHS of (A.2) becomes

$$\begin{aligned} & (D - \triangle A^S)\frac{\overline{L}}{\overline{L}-1} - (A^S - \triangle A^S) \\ = \ & (D\frac{\overline{L}}{\overline{L}-1} - A^S) - \triangle A^S \cdot \frac{1}{\overline{L}-1} \\ < \ & D\frac{\overline{L}}{\overline{L}-1} - A^S. \end{aligned}$$

We can re-write (A.2) in terms of ($E_0$, $L_0$):

$$L_0 < \overline{L} - \frac{v - C(d_B)}{E_0} \cdot (\overline{L} - 1). \tag{A.3}$$

Define $L^*_B(d_B) \equiv \overline{L} - \frac{v-C(d_B)}{E_0} \cdot (\overline{L} - 1)$, which is decreasing in $d_B$. So Lemma 1 is obtained.

**Proof of Lemma 2:** First, additional assumptions that make the problem interesting include:

$\mathbb{E}[\min(\widetilde{X}, KR)] > K$: If no creditor calls the loan at $T_1$, the expected debt payoff for a creditor at $T_2$, which is $\mathbb{E}[\min(\widetilde{X}, KR)]$, is higher than the debt payoff at $T_1$;

$\pi(1) = f - d_H < K$: If all creditors call their loans at $T_1$, FI-H goes bankrupt at $T_1$, because even if FI-H liquidates its whole asset at $T_1$, the revenue generated is not sufficient to cover the face value of the debt;

$f > 2d_H$: This assumption is purely for technical reasons. It makes the revenue function $\pi(s)$ monotonically increasing in $s$ in the interval $s \in [0, 1]$.

Second, we prove Lemma 2. We divide $u$ into two regions: low $u$ and high $u$. In the first region where $u$ is low, FI-H does not need to liquidate its whole asset to repay the early-withdrawing creditors. The staying creditors still obtain a positive payoff at $T_2$. In this case, the payoff of calling the loan at $T_1$ is $K$, while the payoff of holding is $\frac{E[\min((1-s)\widetilde{X},(1-u)KR)]}{1-u} = E[\min(\frac{1-s}{1-u}\widetilde{X}, KR)]$, where $s$ is the proportion of asset that is sold to honor those creditors that decide to withdraw early. Note that $s$ solves $u \cdot K = \pi(s)$. Therefore, $s = \frac{f-\sqrt{f^2-4u\,\mathrm{K}d_H}}{2d_H}$. Using $s = \frac{f-\sqrt{f^2-4u\,\mathrm{K}d_H}}{2d_H}$, we have the payoff function for extending the loan; that is, $w^H(u) = E[\min(\frac{(2d_H-f)+\sqrt{f^2-4u\,\mathrm{K}d_H}}{2(1-u)d_H}\widetilde{X}, KR)]$. In the second region, where $u$ is high, FI-H needs to liquidate its whole asset. The creditors that stay obtain 0, and the calling creditors divide all the liquidation value. Each creditor that calls the loan at $T_1$ obtains $\frac{f-d_H}{u}$. The threshold between the first and the second region is the $u$ that solves $\frac{f-\sqrt{f^2-4u\,\mathrm{K}d_H}}{2d_H} = 1$. The threshold value of $u$ is, after simple computation, equal to $\frac{f-d_H}{K}$.

**Proof of Proposition 1 and Corollary 1:** We proceed in several steps. First, we compute the aggregate proportion of lenders calling loans for a given $L_0$, conditional on every small lender using the threshold $L_S^*$. Given that the small lenders' signals are uniformly distributed in $[L_0 - \epsilon, L_0 + \epsilon]$, the fraction of small lenders calling is $\frac{L_0+\epsilon-L_S^*}{2\epsilon}$. We have the aggregate calling function:

$$u(L_0) = \begin{cases} 0 & L_0 < L_S^* - \epsilon \\ (1-\lambda)\frac{L_0+\epsilon-L_S^*}{2\epsilon} & L_S^* - \epsilon \le L_0 < L_B^* \\ \lambda + (1-\lambda)\frac{L_0+\epsilon-L_S^*}{2\epsilon} & L_B^* \le L_0 \le L_S^* + \epsilon \\ 1 & L_0 > L_S^* + \epsilon \end{cases}, \tag{A.4}$$

where $u(L_0)$ is the aggregate proportion of lenders (large and small lenders) calling for a given $L_0$. There is a discrete jump at $L_0 = L_B^*$, because the large lender also calls when $L_0 = L_B^*(d_B)$. Figure A-1 depicts $u(L_0)$.

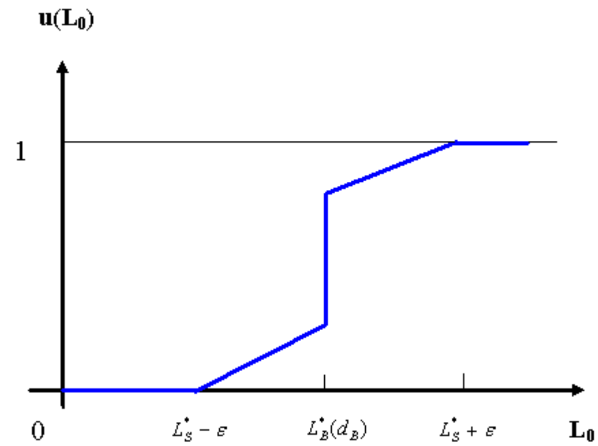


**Figure A-1**: Proportion of lenders withdrawing

Second, we consider the position of the *marginal* small lender that receives the signal $L_S^*$. Since the payoff for a lender is a function of $u$, we wonder what the distribution of $u$ is in the eyes of the marginal lender. From the improper prior distribution of $L_0$, the posterior density of $L_0$ in the eyes of the marginal lender is uniform over the interval $[L_S^* - \epsilon,\ L_S^* + \epsilon]$. Because $u$ is a function of $L_0$, the density of $u$ in the marginal lender's eyes is given by

$$g(u) = \begin{cases} \frac{1}{1-\lambda} & u \in [0,\ (1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon}\ ] \cup [\lambda + (1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon},\ 1] \\ 0 & u \in ((1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon},\ \lambda + (1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon}) \end{cases} . \tag{A.5}$$

The discontinuity in the support of $g(u)$ comes from the jump in the function given by (A.4). Figure A-2 depicts the density $g(u)$.

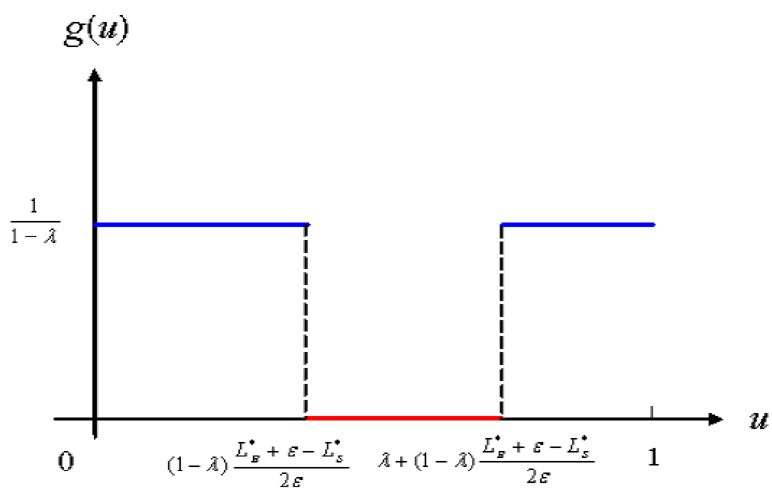


**Figure A-2**: Density of $g(u)$

Third, since the marginal small lender is indifferent to holding versus calling, the expectation of his net payoff from holding (versus calling) is 0:

$$\int_0^1 \triangle w(u) \cdot g(u) du = 0. \tag{A.6}$$

From (A.6), we obtain the unique $L_S^*$.

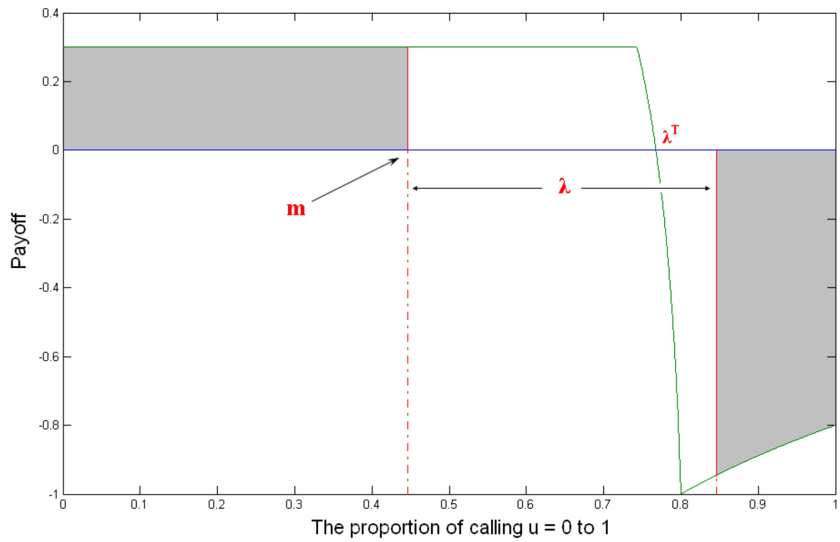


**Figure A-3**: Unique equilibrium

Importantly, equation (A.6) gives a geometrical presentation of finding the equilibrium. In the $\triangle w - u$ space in Figure A-3, the equilibrium means that a (starting) point in the $u$-axis needs to be located such that if we cut a $\lambda$-width block horizontally starting from that point, the area of the remaining part above the $u$-axis (i.e., the shaded area above) should be equal to the area below the $u$-axis (i.e., the shaded area below). We denote the $u$-coordinate of this starting point by $m$, which is a deterministic function of $K$, $R$, $f$, $d_H$ and $\lambda$. Formally, $m$ solves the equation

$$\int_0^m \triangle w(u) du + \int_{m+\lambda}^1 \triangle w(u) du = 0.$$

From (A.4), we also know that $m = (1-\lambda)\frac{L_B^* + \epsilon - L_S^*}{2\epsilon}$. Therefore, we obtain the equilibrium threshold:

$$L_S^* = L_B^*(d_B) + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m(K, R, f, d_H, \lambda). \tag{A.7}$$

Fourth, we consider the condition for the unique equilibrium. From the proof of Proposition 1, $\lambda$ should be sufficiently large to guarantee that a starting point $m$ exists. Figure A-4 illustrates how to calculate the minimum $\lambda$ that guarantees the existence of an equilibrium. The minimum $\lambda$, denoted by $\underline{\lambda}$, is the width of the cutting block such that the sum of the remaining area, after cutting, is zero (i.e., the two shaded areas cancel each other). In fact, when $\lambda \to 0$, the bank run converges to the outcome in Diamond and Dybvig (1983), where multiple equilibria exist.

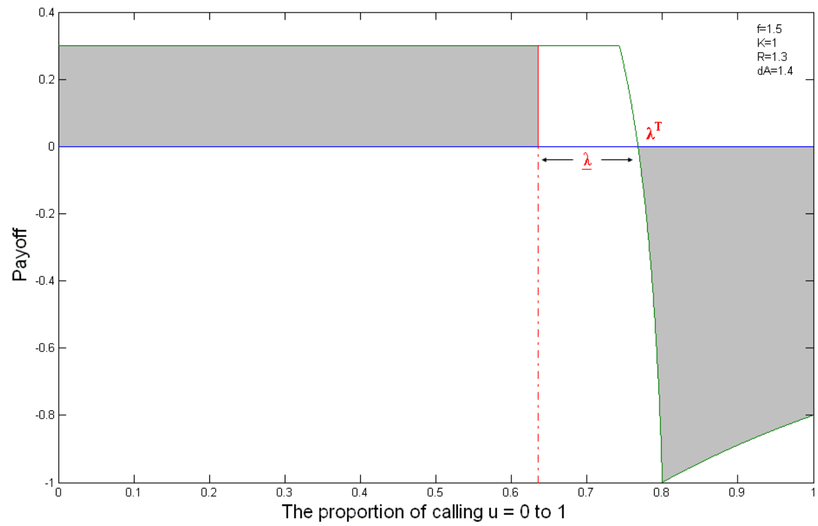


**Figure A-4**: Existence of equilibrium

Fifth, we have Corollary 1. From the proof above, we see that $m$ *is independent of the threshold* $L_B^*$. That is, no matter what threshold strategy the large lender uses, given the large lender's threshold strategy, the equilibrium among the small lenders results in a proportion $m$ of them withdrawing before the large lender does.

Finally, to complete the proof, we need to show that a small lender prefers to call (hold) if its signal $L_0^i$ is higher (lower) than $L_S^*$. The proof is similar to that in Goldstein and Pauzner (2005). In our model, $\triangle w(u)$ is not a strictly monotonically decreasing function of $u$. But it satisfies the 'single crossing property'. In fact, if $L_0^i > L_S^*$, the density $g(u)$ in the small investor $i$'s eyes can be obtained from the original density function (A.5), by transferring weight from the interval $[0, (1-\lambda)\frac{L_0^i - L_S^*}{2\epsilon}]$ to the atom on point $u = 1$. Geometrically, this is equivalent to taking a slice from the positive part of the shaded area above the $u$-axis in Figure A-3 and adding it to the negative part of the shaded area. So the sum of shaded areas becomes less than 0, which means holding (versus calling) has a negative payoff. Calling is then the optimal strategy. A similar argument applies in the opposite case: $L_0^i < L_S^*$.

**Proof of Proposition 2:** Suppose that the large lender uses a strategy different from the one in Lemma 1. Clearly, due to the risk management requirement, FI-B cannot use a threshold high than $L_B^*(d_B)$. It can only use a lower threshold. Let the new threshold be $L_B^{**}$, where $L_B^{**} < L_B^*(d_B)$.

Given that small lenders use the strategies in Proposition 1, the aggregate proportion of calling $u(L_0)$ in (A.4) when the large lender sets its threshold as $L_B^*$ versus $L_B^{**}$ is plotted as follows:

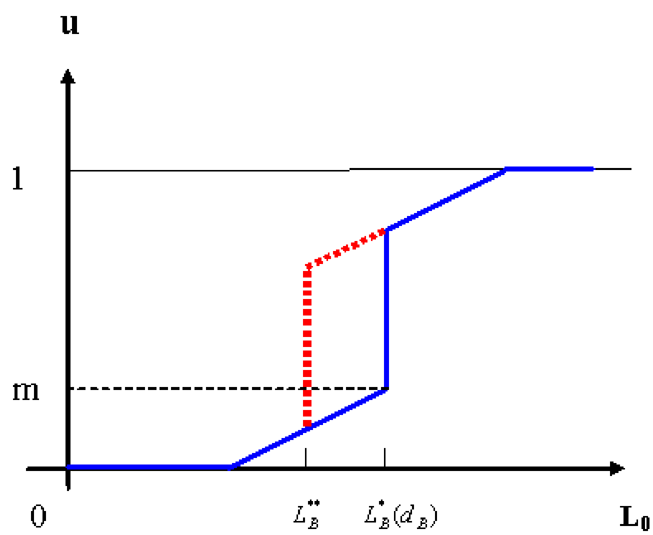


**Figure A-5**: Proportion of lenders withdrawing under an alternative strategy

From Figure A-5, the large lender's action is different under the two thresholds only when $L_B^{**} < L_0 < L_B^*(d_B)$. Therefore, we focus on the discussion in the interval $L_0 \in (L_B^{**}, L_B^*(d_B))$. Under the original threshold $L_B^*(d_B)$, the large lender rolls over the loan. We can conclude that under the original threshold the large lender's payoff is at least equal to $w^H(m)$ because $u < m$ as seen in Figure A-5 and $w^H(\cdot)$ is non-increasing. Under the new threshold $L_B^{**}$, the large lender calls. Its payoff is determined by the function $w^C(\cdot)$. Because of $\lambda \geq \underline{\lambda}(K, R, f, d_H)$, we have $m < \lambda^T$, which is shown in Figure A-3, where $\lambda^T$ solves $\triangle w(\lambda^T) = 0$ or $w^H(\lambda^T) = w^C(\lambda^T)$. So, $w^H(m) > w^H(\lambda^T)$ by the property that $w^H(u)$ is decreasing when $u \leq \lambda^T$. Furthermore, as shown in Figure 2(a), we have $w^C(\lambda^T) \geq w^C(u)$ for any $u \in [0, 1]$. Therefore, by combining $w^H(m) > w^H(\lambda^T)$, $w^H(\lambda^T) = w^C(\lambda^T)$, and $w^C(\lambda^T) \geq w^C(u)$, we have $w^H(m) > w^C(u)$ for any $u \in [0, 1]$. We can then conclude that the payoff to the large lender is always higher by using the threshold $L_B^*(d_B)$ than by using the threshold $L_B^{**}$. The large lender has no incentive to use a strategy different from that in Lemma 1.

**Proof of Proposition 3:** Consider that FI-B uses the threshold $L_B^{*'} = \overline{L} - \frac{v - C'(d_B)}{E_0} \cdot (\overline{L} - 1)$ and small lenders use the threshold $L_S^{*'} = L_B^{*'}(d_B) + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m(K,\ R,\ f,\ d_H,\ \lambda)$. We prove that these strategies are the best responses to each other. By (A.7), we know that the strategy of small lenders is the best response to that of FI-B. Now we prove that the opposite is true.

We fix $L_0$. Given the fixed $L_0$, we define a cut-off $d_B^*$ such that $L_B^{*'}|_{d_B = d_B^*} = L_0$. By the definition of $L_B^{*'}$, the risk management constraint is just binding for the leverage level $L_0$ when the asset value follows the distribution $A_{1+}^L \sim N(v - d_B^* \cdot s_1, \sigma^2 d_B^{*2})$.

We prove that when FI-B observes $d_B < d_B^*$ it holds the loan, and when $d_B \geq d_B^*$ it calls. Given the small lenders' strategy and $L_0$, when $d_B < d_B^*$, there are less than $m$ proportion of small lenders withdrawing. So the selling from the FI-H is less than $s_1$. Let the selling amount be $s_1^{'}$, where $s_1^{'} < s_1$. Therefore, the asset value at $T_{1+}$ is distributed as $A_{1+}^L \sim N(v - d_B \cdot s_1^{'}, \sigma^2 d_B^2)$. Considering $d_B < d_B^*$ and $s_1^{'} < s_1$, this implies that the mean of the distribution increases while the variance decreases, relative to the distribution $A_{1+}^L \sim N(v - d_B^* \cdot s_1, \sigma^2 d_B^{*2})$. The two joint forces lead to the risk management constraint certainly not binding for the leverage level $L_0$. Therefore, FI-B holds. Similarly, we can prove that FI-B calls when $d_B \geq d_B^*$.

Now we will prove the uniqueness of the equilibrium. As in the above analysis, we obtain that the only strategy that the large lender chooses to play is the one with the threshold being $L_B^{*'} = \overline{L} - \frac{v - C'(d_B)}{E_0} \cdot (\overline{L} - 1)$. It is not optimal for FI-B to use a different threshold strategy. In fact, if it uses a threshold lower than $L_B^{*'}$, it loses the interest $R$ (in some states); if it uses a threshold higher than $L_B^{*'}$, its risk management requirement is not satisfied.

**Proof in Section 2.3.1:** Consider $L_S^* = L_B^*(d_B) + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m(K,\ R,\ f,\ d_H,\ \lambda)$. Since $\frac{\partial L_B^*(d_B)}{\partial d_B} < 0$, we have $\frac{\partial L_S^*(d_H, d_B)}{\partial d_B} < 0$.

In general, we have $\frac{\partial m(K,R,f,d_H,\lambda)}{\partial d_H} > 0$ and thus $\frac{\partial L_S^*(d_H, d_B)}{\partial d_H} < 0$. Intuitively, if $d_H$ increases, the liquidation value decreases. Hence, in Figure A-3, the shaded area below expands, while the shaded area above remains the same, which means that the original equilibrium is not valid. In the new equilibrium, the two shaded areas need to cancel out. So the shaded area above needs to expand. That is, $m$ has to move to the right and increase. As $m(K, R, f, d_H, \lambda)$ has no closed-forms, we have to rely on simulation. The simulation result confirms this. Note that the comparative static result that $\frac{\partial m}{\partial d_H} > 0$ is not always true because the payoff structure of a bank run is not of *strict* strategic complementarities. However, we are interested in the general cases. For a wide range of parameter values, we have $\frac{\partial m}{\partial d_H} > 0$.

Consider $L_B^{*'} = \overline{L} - \frac{v - C'(d_B)}{E_0} \cdot (\overline{L} - 1)$. As $C'$ is the $\alpha$-percentile of the distribution $A_{1+}^L \sim N(v - d_B \cdot s_1, \sigma^2 d_B^2)$, we have $\frac{\partial^2 C'}{\partial s_1 \partial d_B} < 0$. Since $\frac{\partial m(K,R,f,d_H,\lambda)}{\partial d_H} > 0$ and $s_1$ is increasing in $m$, then $\frac{\partial s_1}{\partial d_H} > 0$. Thus, $\frac{\partial^2 C'}{\partial d_H \partial d_B} < 0$. Therefore, $\frac{\partial^2 L_B^{*'}}{\partial d_H \partial d_B} < 0$. Also, because $L_S^{*'} = L_B^{*'} + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m$, we have $\frac{\partial^2 L_S^{*'}}{\partial d_H \partial d_B} < 0$.

**Proof of Proposition 5:** By (A.4), we have $m = (1-\lambda)\frac{L_B^* + \epsilon - L_S^*}{2\epsilon}$. From (8), we obtain

$$\frac{L_B^* - L_S^* + \epsilon}{2\epsilon} = \frac{-\int_\lambda^1 \triangle\, w(u)du}{-\int_\lambda^1 \triangle\, w(u)du + \int_0^{1-\lambda} \triangle\, w(u)du}.$$

Hence, $m = (1-\lambda)\frac{-\int_\lambda^1 \triangle w(u)du}{-\int_\lambda^1 \triangle w(u)du + \int_0^{1-\lambda} \triangle w(u)du}$.

**Proof of Proposition 6:** The proof is similar to the proof of Proposition 5.

**Proof of Corollary 2:** The $m$ in Corollary 2 can be obtained in a similar way to the proof of Proposition 5. We turn to comparative statics. Because $\triangle\hat{w}(u)$ and $\triangle\bar{w}(u)$ are non-monotonic, we cannot show the monotonicity of $m$ in the whole domain of $\kappa$. We focus on the interesting case such that $m$ is increasing in $\kappa$. Note that when $\kappa$ is sufficiently high, $\int_0^\kappa \triangle\bar{w}(u)du < 0$. There exists $\kappa$ such that $\frac{|\bar{w}(\kappa)|}{\hat{w}(\kappa)} > \frac{\left|\int_0^\kappa \triangle\bar{w}(u)du\right|}{\int_0^\kappa \triangle\hat{w}(u)du}$. Under this condition, $\frac{\hat{m}}{1-\hat{m}} = \frac{\left|\int_0^\kappa \triangle\bar{w}(u)du\right|}{\int_0^\kappa \triangle\hat{w}(u)du}$ is increasing $\kappa$, where $\hat{m} = \frac{m}{1-\lambda}$; therefore, $m$ is increasing in $\kappa$. So $L_S^*$ is decreasing in $\kappa$.

**Proof of Corollary 3:** The $m$ in Corollary 3 can be obtained in a similar way to the proof of Proposition 5. We turn to comparative statics. Note that $\int_0^{1-\lambda} \triangle\bar{w}(u)du < 0$. Clearly, $m$ is decreasing in $\delta$, so $L_S^*$ is increasing in $\delta$.

**Proof in Section 3.3:** By (A.3), $0 < L_0 - 1 < (\overline{L} - 1) \cdot \left[1 - \frac{v - C(d_B)}{E_0}\right]$. So $1 - \frac{v - C(d_B)}{E_0} > 0$. Considering $L_B^*(d_B) - 1 \equiv \left(\overline{L} - 1\right) \cdot \left[1 - \frac{v - C(d_B)}{E_0}\right]$, it is easy to show that $\frac{\partial L_B^*}{\partial \alpha} > 0$ and $\frac{\partial L_B^*}{\partial \overline{L}} > 0$.

**Proof of Lemma 3:** Equations (13) and (14) can be rewritten as (A.8) and (A.9), respectively:

$$\int_{\theta=x^*-\sigma G^{-1}(\frac{\frac{L}{K}}{1-\lambda})}^{\theta=x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})} (KR\cdot\theta)\cdot(1-q)H(\frac{\theta-y^*}{\delta})dG(\frac{\theta-x^*}{\sigma}) + \int_{\theta=x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})}^{\theta=+\infty} (KR\cdot\theta)dG(\frac{\theta-x^*}{\sigma}) = K, \tag{A.8}$$

$$\int_{\theta=x^*-\sigma G^{-1}(\frac{\frac{L}{K}}{1-\lambda})}^{\theta=+\infty} (KR\cdot\theta)dH(\frac{\theta-y^*}{\delta}) = K. \tag{A.9}$$

We denote the LHS of (A.8) by $F^s(x^*;y^*)$, which is a function of $x^*$ with parameter $y^*$. Similarly, the LHS of (A.9) is denoted by $F^l(y^*;x^*)$, which is a function of $y^*$ with parameter $x^*$.

First, we prove that equation (A.8) has a unique solution with respect to $x^*$ for a given $y^*$. To show this, we prove that $F^s(x^*;y^*)$ is monotonically increasing in $x^*$. That is, $F^s(x^*+\triangle;y^*) > F^s(x^*;y^*)$ for any $x^*$ and $\triangle > 0$. In fact,

$$\begin{aligned}
&F^s(x^*+\triangle;y^*)\\
&= \int_{\theta=(x^*+\triangle)-\sigma G^{-1}(\frac{\frac{L}{K}}{1-\lambda})}^{\theta=(x^*+\triangle)-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})} \frac{KR\cdot\theta}{\frac{1}{(1-q)H(\frac{\theta-y^*}{\delta})}}\cdot dG(\tfrac{\theta-(x^*+\triangle)}{\sigma}) + \int_{\theta=(x^*+\triangle)-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})}^{\theta=+\infty} (KR\cdot\theta)dG(\tfrac{\theta-(x^*+\triangle)}{\sigma})\\
&= \int_{\theta'=x^*-\sigma G^{-1}(\frac{\frac{L}{K}}{1-\lambda})}^{\theta'=x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})} \frac{KR\cdot(\theta'+\triangle)}{\frac{1}{(1-q)H(\frac{(\theta'+\triangle)-y^*}{\delta})}}\cdot dG(\tfrac{\theta'-x^*}{\sigma}) + \int_{\theta'=x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})}^{\theta'=+\infty} (KR\cdot(\theta'+\triangle))dG(\tfrac{\theta'-x^*}{\sigma})\\
&> \text{LHS of (A.8)}\\
&= F^s(x^*;y^*).
\end{aligned}$$

The third line above is obtained when we substitute $\theta' = \theta - \triangle$ into the second line. Further, it is easy to show that $F^s(x^*;y^*) < K$ when $x^*$ is very low and $F^s(x^*;y^*) > K$ for a very high $x^*$. Therefore, $F^s(x^*;y^*) = K$ has a unique solution.

We denote the unique solution to (A.8) by $x^*(y^*)$. We have the following properties regarding $x^*(y^*)$: The solution $x^*(y^*)$ is increasing in $y^*$ and has a bounded range denoted by $(\underline{x^*}, \overline{x^*})$, where $\underline{x^*}$ is the lower bound and $\overline{x^*}$ is the upper bound. In fact, it is easy to show that $\frac{\partial F^s(x^*;y^*)}{\partial y^*} < 0$ (for $q < 1$).[38] Combining this with $\frac{\partial F^s(x^*;y^*)}{\partial x^*} > 0$, we have $\frac{dx^*(y^*)}{dy^*} = -\frac{\frac{\partial F^s(x^*;y^*)}{\partial y^*}}{\frac{\partial F^s(x^*;y^*)}{\partial x^*}} > 0$. As

[38] When $q = 1$, the system of equations (A.8) and (A.9) is reduced to equation (A.8), which clearly has a unique

for the boundness, in (A.8), when $y^* = -\infty$ (respectively $+\infty$), it follows that $H(\frac{\theta - y^*}{\delta}) = 1$ (respectively 0) and therefore (A.8) admits bounded solutions. That is, $x^*(y^*)|_{y^*=-\infty} = \underline{x^*} > -\infty$ and $x^*(y^*)|_{y^*=+\infty} = \overline{x^*} < +\infty$.

Second, it is easy to prove that equation (A.9) has a unique solution with respect to $y^*$ for a given $x^*$. We denote the unique solution by $y^*(x^*)$. It also follows that $y^*(x^*)$ is an increasing function with the slope $\frac{dy^*(x^*)}{dx^*} < 1$. In fact, if both $x^*$ and $y^*$ increase by same amount in (A.9), the RHS would exceed the left hand side; so $y^*$ has to increase less than $x^*$ in order to keep the equality.

Third, now we are able to prove that the system of equations (A.8) and (A.9) has solutions. This result follows from the fact that $x^*(y^*)$ and $y^*(x^*)$ are both increasing functions and $x^*(y^*)$ is bounded within $(\underline{x^*}, \overline{x^*})$. So the fixed point problem has a unique solution. In fact, by (A.9), we have $y^*(x^*)|_{x^*=\underline{x^*}} > -\infty$ and $y^*(x^*)|_{x^*=\overline{x^*}} < +\infty$. Combining this with $x^*(y^*)|_{y^*=-\infty} = \underline{x^*}$ and $x^*(y^*)|_{y^*=+\infty} = \overline{x^*}$, we conclude that the two curves of $x^*(y^*)$ and $y^*(x^*)$ interact and hence the system of equations has solutions.

Finally, we prove the uniqueness of solutions. We prove by contradiction. Suppose there exist two pairs of solutions for the system of equations (A.8) and (A.9). Let them be ($x^*$, $y^*$) and ($x^{*'}$, $y^{*'}$), where $x^* < x^{*'}$ and $y^* < y^{*'}$. Denote $\triangle = x^{*'} - x^*$. By the property $\frac{dy^*(x^*)}{dx^*} < 1$ derived from (A.9) shown above, we have $y^{*'} - y^* < \triangle$. Now we check (A.8) and prove that equation (A.8) is incompatible with the two pairs of solutions with $x^{*'} = x^* + \triangle$ and $y^{*'} < y^* + \triangle$. In fact,

$$
\begin{aligned}
& F^s(x^{*'}; y^{*'}) \\
& > F^s(x^* + \triangle; y^* + \triangle) \\
& = \int_{\theta=(x^*+\triangle)-\sigma G^{-1}(\frac{\frac{L}{K}}{1-\lambda})}^{\theta=(x^*+\triangle)-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})} \frac{(KR\cdot\theta)\cdot(1-q)}{\frac{1}{H(\frac{\theta-(y^*+\triangle)}{\delta})}} dG(\tfrac{\theta-(x^*+\triangle)}{\sigma}) + \int_{\theta=(x^*+\triangle)-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})}^{\theta=+\infty} (KR\cdot\theta) dG(\tfrac{\theta-(x^*+\triangle)}{\sigma}) \\
& = \int_{\theta'=x^*-\sigma G^{-1}(\frac{\frac{L}{K}}{1-\lambda})}^{\theta'=x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})} \frac{(KR\cdot(\theta'+\triangle))\cdot(1-q)}{\frac{1}{H(\frac{\theta'-y^*}{\delta})}} dG(\tfrac{\theta'-x^*}{\sigma}) + \int_{\theta'=x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})}^{\theta'=+\infty} (KR\cdot(\theta'+\triangle)) dG(\tfrac{\theta'-x^*}{\sigma}) \\
& > \text{LHS of (A.8)} \\
& = K.
\end{aligned}
$$

solution by plugging $q = 1$.

The second line above follows because $\frac{\partial F^s(x^*;y^*)}{\partial y^*} < 0$. The fourth line is obtained when we apply $\theta' = \theta - \triangle$ in the third line.

To conclude, the system of equations (A.8)-(A.9) has a unique solution and therefore there is a unique threshold equilibrium for the creditor-run game.

**Proof of Lemma 4:** As in Corsetti et al. (2004), we focus on the limit case where $\delta \to 0$, $\sigma \to 0$, and $\frac{\delta}{\sigma} \to c$. The limit case is used for tractability. In the limit, FI-H fails at $T_1$ if and only if $\theta \le \theta^*$, where $\theta^*$ is the cutoff denoting FI-H's failure. First, as $\delta \to 0$ and $\sigma \to 0$, it follows that $x^* = y^*$, meaning that either all lenders (large and small) or none decides to withdraw. Clearly, when FI-B is not *actually* constrained at $T_1$, FI-H fails if and only if $\theta \le \theta^*$ where $\theta^* = x^* = y^*$. Second, when FI-B is *actually* constrained, FI-H still fails if and only if $\theta \le \theta^*$ where $\theta^* = x^*$. In fact, if $\theta > x^*$, all the small lenders stay on although the large lender is constrained and has to run, which, however, is not sufficient to cause the failure of FI-H by recalling $\lambda \le \frac{L}{K}$. Therefore, overall, the failure threshold is $\theta^* = x^* = y^*$ and, in particular, the failure threshold is not contingent on whether FI-B is actually constrained at $T_1$.

1. We consider two polar cases: $q = 0$ and $q = 1$.

**Case 1**: $q = 0$ (FI-B is not constrained)

Because $\sigma \to 0$, equations (13) and (14) can be transformed into (A.10) and (A.11), respectively.

$$\underbrace{\left\{\frac{\frac{L}{K}}{1-\lambda} \cdot H\left(\frac{x^*-y^*}{\delta}\right) + \frac{\frac{L}{K}-\lambda}{1-\lambda}\left[1 - H\left(\frac{x^*-y^*}{\delta}\right)\right]\right\}}_{\text{Coordination risk}} \underbrace{(KR \cdot x^*)}_{\text{Fundamental risk}} = K \tag{A.10}$$

$$\int_{\theta=x^*}^{\theta=+\infty} (KR \cdot \theta) dH\left(\frac{\theta - y^*}{\delta}\right) = K. \tag{A.11}$$

Intuitively, in (A.10) the coordination risk is the weighted average of the coordination risk when FI-B definitely does not run and when it definitely runs.

We will now prove that the solution of $x^*$ to the system of equations (A.10) and (A.11) when $\lambda > 0$ is lower than when $\lambda = 0$. First, we look at $\lambda = 0$. In this case, (A.10) becomes

$$\underbrace{\frac{\frac{L}{K}}{1}}_{\text{Coordination risk}} \cdot \underbrace{(KR \cdot x^*)}_{\text{Fundamental risk}} = K \tag{A.12}$$

Second, we examine the solution $x^*$ when $\lambda > 0$. We consider an alternative equation to (A4):

$$\int_{\theta=x^*}^{\theta=+\infty} (KR\cdot x^*)dH(\frac{\theta-y^*}{\delta})=K \tag{A.13}$$

We can verify that the system of equations of (A.10) and (A.13) has the exact same solution of $x^*$ as in (A.12). In fact, (A.13) can be transformed to $(KR\cdot x^*)(1-H(\frac{x^*-y^*}{\delta}))=K$. So the equation system (A.10) and (A.13) has a unique solution which satisfies (A.12) and $H(\frac{x^*-y^*}{\delta})=1-\frac{L}{K}$. Now we examine the solution to the system of equations (A.10) and (A.11). Note that by comparing the LHS of (A.11) and (A.13), we have $\int_{\theta=x^*}^{\theta=+\infty}(KR\cdot\theta)dH(\frac{\theta-y^*}{\delta})>\int_{\theta=x^*}^{\theta=+\infty}(KR\cdot x^*)dH(\frac{\theta-y^*}{\delta})$. Therefore, the unique solution to (A.10) and (A.11) must have the following properties: $x^*$ is lower than that in (A.12) and the solution satisfies $H(\frac{x^*-y^*}{\delta})>1-\frac{L}{K}$. In fact, we can obtain the unique solution to (A.10) and (A.11) by iteration. In the first step, we set the solution to (A.10) and (A.11) as the initial trial. Clearly, with this trial, (A.11) is not satisfied, given that $x^*$, $y^*$ has to decrease. Then go to (A.10); because $y^*$ goes down and thus $H\left(\frac{x^*-y^*}{\delta}\right)$ goes up (for a given $x^*$), $x^*$ must decrease to keep (A.10) valid. The decrease in $x^*$ further triggers the decrease in $y^*$ in (A.10), and so on.

By $\sigma\to 0$, we have $\theta^*=x^*$. Therefore, $\theta^*$ is lower when $\lambda>0$ than when $\lambda=0$.

Now we prove that $\theta^*$ is decreasing in $\lambda$ when $\lambda$ is big enough. We examine equation (A.8) by substituting $q=0$. When $\lambda$ increases, there are two effects. First, the lower bound of integral, $x^*-\sigma G^{-1}(\frac{\frac{L}{K}}{1-\lambda})$, decreases, which has a positive effect on the LHS of (A.8). Second, the integral bound $x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})$ increases, which has a negative effect on the LHS of (A.8). The second effect is negative because $KR\cdot\theta>(KR\cdot\theta)\cdot H(\frac{\theta-y^*}{\delta})$. However, the second effect is diminishing when $\lambda$ increases. In fact, $1-H(\frac{\theta-y^*}{\delta})|_{\theta=x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})}=1-H(\frac{x^*-\sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})-y^*}{\delta})$ is decreasing in $\lambda$. Overall, when $\lambda$ is big enough, an increase in $\lambda$ raises the LHS of (A.8). Therefore, in order to restore the equality, $x^*$ needs to decrease. That is, $x^*$ is a decreasing function of $\lambda$. Under $\sigma\to 0$, $\theta^*$ is also decreasing in $\lambda$.

**Case 2**: $q=1$ (FI-B is definitely constrained)

Because $q=1$, the small lenders' problem in equation (13) is rewritten as:

$$\int_{-\infty}^{+\infty}\left[I(\Phi(\theta;x^*)(1-\lambda)+\lambda\le\frac{L}{K})\cdot(KR\cdot\theta)\right]g(\theta|x^*)d\theta=K. \tag{A.14}$$

As shown in the global games literature, in the limit $\sigma\to 0$, the small lenders have no uncertainty about the fundamentals $\theta$. However, no matter how small $\sigma$ is, they face *strategic uncertainty*. In the eyes of the marginal small lender whose signal is $x^*$, the aggregate fraction of small lenders withdrawing, $\Phi(\theta; x^*)$, is always uniformly distributed within $[0,1]$. So, equation (A.14) becomes:

$$\underbrace{\frac{\frac{L}{K}-\lambda}{1-\lambda}}_{\text{Coordination risk}} \cdot \underbrace{(KR\cdot x^*)}_{\text{Fundamental risk}} = K. \tag{A.15}$$

Solving equation (A.15) gives

$$x^* = \frac{1}{R}\frac{1-\lambda}{\frac{L}{K}-\lambda}. \tag{A.16}$$

Since $\sigma \to 0$, it follows that $\theta^* = x^*$, and $\theta^*$ increases with $\lambda$. That is, for $q=1$, in the limit as $\delta \to 0$, FI-H's failure threshold $\theta^*$ is increasing in $\lambda$.[39]

2. We consider general cases of $0 \le q \le 1$

In the proof of Lemma 3, we have shown that $\frac{\partial x^*}{\partial y^*} > 0$ in (A.8), and that $\frac{\partial y^*}{\partial x^*} \ge 0$ in (A.9). By (A.8), we can also obtain that $\frac{\partial x^*}{\partial q} = -\frac{\frac{\partial F^s}{\partial q}}{\frac{\partial F^s}{\partial x^*}} > 0$.

In the limit as $\delta \to 0$, $\sigma \to 0$, and $\frac{\delta}{\sigma} \to c$, we have $\theta^* = x^*$. So, $\theta^*(\lambda; q)$ is increasing in $q$ (point-wisely).

The function $\theta^*(\lambda; q)$ is continuous in both $\lambda$ and $q$. It is decreasing in $\lambda$ when $q=0$ (under a wide set of parameters) and increasing in $\lambda$ when $q=1$. By continuity and $\theta^*(\lambda; q)$ increasing in $q$ point-wisely, we conclude that $\theta^*(\lambda; q)$ is increasing in $\lambda$ when $q$ is sufficiently close to 1, and decreasing in $\lambda$ when $q$ is sufficiently close to 0.

**Proof of Proposition 7:** In the close set $\{(\lambda, q)|\lambda \in [0, \min\{\frac{L}{K}, 1-\frac{L}{K}\}],\, q \in [0,1]\}$ function $\theta^*(\lambda; q)$ is continuous. So the optimization problem in (15) has solutions in the set $\lambda \in [0, \min\{\frac{L}{K}, 1-\frac{L}{K}\}]$ . Under some parameter values and probability distribution $z(q)$, it has a unique interior solution (i.e., the optimal $\lambda$ lies in $(0, \min\{\frac{L}{K}, 1-\frac{L}{K}\})$). The numerical example in the text is one case. In fact, $\theta^*(\lambda; q)$ is convex in $\lambda$ and increases very fast with $\lambda$ when $q=1$. So if the probability for $q=1$ is not too low, it is not optimal to choose a very high $\lambda$. On the other hand, if the probability for $q=1$ is sufficiently high, it is not optimal to choose $\lambda=0$. Overall, for a certain distribution $z(q)$, the optimal $\lambda$ is an interior solution.

Also, under some parameter values and probability distribution $z(q)$, it is not optimal to choose a $\lambda$ above $\min\{\frac{L}{K}, 1-\frac{L}{K}\}$. In fact, if $\lambda$ is too large, $\theta^*$ is very high in the case of $q=1$. In particular,

[39] For $q=1$, under a general $\sigma > 0$ away from the limit, we can still prove that $x^*$ and $\theta^*$ increase with $\lambda$. In fact, based on the proof of Lemma 3, when $q=1$, (A.8) has a unique solution $x^*$ which is increasing in $\lambda$. Also, when $q=1$, we have $\theta^* = x^* - \sigma G^{-1}(\frac{\frac{L}{K}-\lambda}{1-\lambda})$; so $\theta^*$ is certainly increasing in $\lambda$.

when $\lambda$ is close to $\frac{L}{K}$, $\theta^*$ approaches $+\infty$ in the case of $q = 1$ by (A.16).

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