

Credit Expansion and Credit Misallocation^{*†}

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Abstract

This paper develops a general equilibrium model to analyze the interaction between two sectors with differing degree of financial friction in the context of liquidity injections (credit expansion). We show that if too much liquidity is injected into the economy, overheating can build up in the sector with lower friction, crowding liquidity out of the sector with higher friction. The crowding-out manifests in a self-reinforcing spiral because of feedback between liquidity inflows, asset prices, and collateral values. The paper highlights the effect of financial frictions on the allocation and distribution of liquidity in an economy, demonstrating misallocation of liquidity (credit) in the economy under excessive liquidity injections.

JEL classification: G01; G21; G33; E58

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1 Introduction

Misallocation of credit (or capital) can be a critical reason for underperformance of the aggregate economy (Restuccia and Rogerson (2008), Hsieh and Klenow (2009)). Importantly, misallocation of credit is often particularly severe following large-scale capital inflows or credit expansion for an economy. For example, the financial integration in the EU since the late 1990s led to massive foreign credit pouring into the EU’s peripheral countries. The capital inflow, however, resulted in relatively unproductive firms in the nontradables sector expanding at the expense of more productive tradables firms (see Reis (2013)). In the same spirit, in response to the 2007-2009 global financial crisis, the Chinese government implemented some aggressive credit expansion policies in an attempt to save the real economy from recession. The result, however, was a surge in real estate prices while small and medium-sized businesses in the real economy had an even harder time obtaining corporate liquidity. Notably, the mortgage-asset bubble that crowded out commercial loans and triggered the 2007-2009 crisis in the U.S. was also rooted in an environment of massive liquidity injections (see the evidence in Chakraborty, Goldstein and MacKinlay (2013)).¹

We present a new channel for misallocation across sectors, arising from the interaction of capital inflows with differing degree of credit friction across sectors. In the model, financial intermediaries (“banks”) lend to entrepreneurs, but due to the financial contracting friction can only lend to them up to the value of their collateral. Hence, lending is limited. The value of the collateral, given by the asset re-sale value in the secondary market, is in turn endogenously determined by the economic activity, which depends on the amount that is lent to entrepreneurs and invested by them. When the government conducts credit expansion by injecting liquid funds into the banking system, this stimulus can ignite a self-reinforcing cycle: the liquidity injection increases lending, which increases investment, which increases collateral prices, which then enables more lending and investment, and so on. In other words, there is a feedback loop between bank lending, secondary-market asset prices, and collateral values.

We then introduce into the economy two sectors, Sectors 1 and 2, with differing degree of financial friction in secondary-market trading. Specifically, the two sectors differ in asset specificity (Williamson (1985, 1986)), which determines margin financing (i.e., asset leverage) in the secondary market. Lower asset specificity enables higher asset leverage. Hence, the difference in financial friction, which originates in asset specificity, results in heterogeneity in secondary-market trading activities across sectors. This heterogeneity causes collateral prices to respond asymmetrically to liquidity injections across the two sectors. The price in the sector with lower asset specificity (Sector 2 in our model) responds more strongly to liquidity injections than that in the sector with higher

¹In fact, in response to the bursting of the Internet bubble in early 2000, the Federal Reserve Bank had adopted a policy of unprecedented credit easing for a prolonged period between 2001 and 2005.

asset specificity (i.e., Sector 1).

This asymmetry across sectors creates a ‘crowding-out’ effect. If too much liquidity is injected into the economy, the asset price and thus the collateral value in Sector 2 can increase so fast that it leads to a rise in the real interest rate in the economy. This occurs because when the collateral value increases, more entrepreneurs qualify for borrowing and thus more of them compete for loans, driving up the interest rate. With the collateral value in Sector 1 not responding much, the higher interest rate in the economy drives down the “discounted” value of the collateral in that sector, crowding out liquidity of Sector 1. The crowding-out manifests in a self-reinforcing spiral as more liquidity entering Sector 2 drives up the interest rate, which leads to additional liquidity leaving Sector 1 (for Sector 2), driving up the interest rate further, and so on. In short, injecting too much liquidity into the economy actually reduces the amount of liquidity entering Sector 1. If, on the other hand, too little liquidity is injected, Sector 1 of course cannot obtain much liquidity. As a result, we show that there exists an optimal level of liquidity injection for the government.

Our paper highlights the effect of financial frictions on the allocation and distribution of liquidity in the economy. In the two-sector economy setting, we show not only that liquidity tends to move to the sector with lower friction (i.e., the allocation effect) but also that the sector with lower friction can suck liquidity out of the other sector (i.e., the crowding-out effect).²

There is a large literature on financial frictions, and our paper follows the seminal work of Kiyotaki and Moore (1997).³ The innovation is that we consider multiple sectors with varying degrees of asset specificity. In this literature, Benmelech and Bergman (2012) build a novel framework for studying the interplay between financing frictions, liquidity, and collateral values. The authors show that the credit easing policy sometimes does not work because additional liquidity injections may not raise firm asset collateral values and thus credit traps can form. Additional liquidity injections in their model are ineffective but harmless to the economy. Our paper adds to this literature in two ways. First, we study the distribution of liquidity in a two-sector economy, and demonstrate the danger of excessive liquidity injections (beyond inflation): excessive liquidity actually hurts the aggregate economy because it causes misallocation of liquidity across sectors. Second, we provide a new micro-foundation for the effect of liquidity injections on asset prices.

Our paper is related to the growing literature on unconventional credit policies.⁴ As Gertler and Kiyotaki (2010) write, “Since these policies are relatively new, much of the existing literature

²Tirole (1985) and Farhi and Tirole (2012) show that bubbles in unproductive assets can crowd out investments in unrelated real assets. In contrast, in our paper, overheating occurs in the productive investment of a less frictioned sector, crowding out investment in a sector with higher friction. See also Miao and Wang (2014, 2015).

³Brunnermeier et al. (2013) provide an excellent recent survey. Another related literature links corporate finance and macroeconomics (e.g., Allen and Gale (2000), Diamond and Rajan (2006), Shleifer and Vishny (2010a,b), Acharya and Viswanathan (2011), Acharya and Naqvi (2012), Chen and Song (2013)).

⁴See, e.g., Reis (2009), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Stein (2012).

is silent about them.” In the comments on Reis’ (2009) paper,⁵ Besley (2009) notes that “it is possible for central banks to distort the allocation of credit, causing excess credit creation in some areas. Thus, it is important to consider the sectoral credit impact as well as the aggregate effects.” Our research addresses this gap by studying a credit policy model with two sectors and showing cross-sectoral distortions under excess credit creation.

The paper is organized as follows. Section 2 lays out the model setup. Section 3 presents the one-sector economy equilibrium. Section 4 studies the two-sector economy. Section 5 conducts several extensions of the model. Section 6 discusses related empirical evidence. Section 7 concludes.

2 Model Setup

Consider an economy with two sectors, labeled Sector 1 and Sector 2. The two sectors differ in asset specificity, which we will elaborate on. Each sector consists of a continuum of self-employed entrepreneur-households of measure one (as in Mendoza (2010)). For ease of exposition, we do not distinguish between the two sectors at this stage. In the economy, there is also a set of competitive financial intermediaries (called “banks”) that supply capital to entrepreneurs, and a government (or policymaker). The model has three dates: T_0 , T_1 and T_2 . There is no time discount between dates.

2.1 Entrepreneurs

Each entrepreneur has an asset *in place* at T_0 — an identical investment project across all entrepreneurs. Entrepreneurs undertook their project before T_0 , which is expected to generate a constant cash flow C at T_1 and a stochastic cash flow \tilde{x} at T_2 , where \tilde{x} has one of two realizations, $\tilde{x} \in \{u, d\}$, and $u > d > 0$.

Only a part of the project’s cash flow is pledgeable. More specifically, the cash flow C is non-pledgeable while a part of the cash flow \tilde{x} is pledgeable. The pledgeable part of \tilde{x} is a constant amount X , where $0 \leq X \leq d$; the remaining part $\tilde{x} - X$ is non-pledgeable. As is standard in the financial contracting literature (e.g., Hart and Moore (1998), Tirole (2010)), the interpretation is the following.

The project’s cash flow is unverifiable and hence uncontractible. In the event that the owner of a project defaults at T_2 , outside investors (i.e., debt-holders) obtain and exercise the control right over the asset; outside investors can only realize a cash flow X when they seize and operate the asset at T_2 due to *asset specificity*. That is, the term X measures asset specificity. The term X

⁵Reis (2009) presents a credit policy model suggesting that providing credit to traders in securities markets can be an effective way of intervention.

equivalently measures firm asset pledgeability or collateralizability at T_2 (Williamson (1988)).

2.2 Liquidity shock

The economy (entrepreneurs) suffers an *unexpected* aggregate liquidity shock at T_0 (as in the business cycle literature, e.g., Kiyotaki and Moore (1997)). That is, an entrepreneur has to invest an *additional* amount I at T_0 to enable his project to deliver the cash flow C , where $I < C$; otherwise his project delivers zero cash flow at T_1 . Delivery of cash flow \tilde{x} will not be affected by the shock.⁶

Entrepreneurs are heterogeneous in internal capital at T_0 . Suppose the amount of internal capital of an entrepreneur at T_0 is A , which means that the entrepreneur needs an amount of external capital, $B \equiv I - A$, to be able to make his liquidity investment. We assume that B has a probability distribution (pdf), $f(B)$, across entrepreneurs, within support $[0, I]$. Let $F(\cdot)$ denote the cumulative distribution function (cdf) of $f(\cdot)$. Clearly, giving the distribution of B is equivalent to giving the distribution of A .

Faced with limited internal capital, entrepreneurs seek to raise external capital by borrowing from banks. The borrowing (debt) is short-term, that is, an entrepreneur needs to repay his debt at T_1 . We will show that long-term debt with maturity T_2 is not optimal or infeasible. Entrepreneurs that do not make the liquidity investment can *deposit* their spare internal capital with banks at T_0 .

It is common knowledge that entrepreneurs will have diverging (heterogeneous) beliefs at T_1 . For simplicity, we assume that there are two types of beliefs at T_1 : high beliefs and low beliefs. High beliefs correspond to $\Pr[\tilde{x} = u] = \theta_H$ and low beliefs to $\Pr[\tilde{x} = u] = \theta_L$, where $\theta_H > \theta_L$. Ex ante, before T_1 , the probability of developing high beliefs for an entrepreneur is π . We also denote the true probability of realizing u of \tilde{x} by θ .

It is realistic to model diverging (heterogeneous) beliefs among entrepreneurs. In fact, in an economic recession, agents are often quite uncertain about economic prospects and have diverging views. In the background of economic stimulus, intensive speculative trade (on commodities, real estates, etc.) under heterogeneous beliefs can occur, which has been frequently observed in many countries.

There is a secondary asset market at T_1 , where entrepreneurs with heterogeneous beliefs trade their assets.⁷ As in Geanakoplos (2010), short-selling is not allowed for the secondary market. In reality, short-selling is either impossible or with constraints.

⁶For simplicity and without loss of generality, we do not explicitly model entrepreneurs' investment and financing decisions prior to T_0 .

⁷We assume that projects are not "mature" enough at T_0 and thus entrepreneurs' assets cannot be traded at T_0 due to the inalienability of human capital (Hart and Moore (1994)).

2.3 Banks

There is a large number of *competitive* financial intermediaries, simply called “banks”, that make loans to entrepreneurs. Denote the net interest rate of bank loans by r . As the cash flow C of an entrepreneur’s project is not contractible, the only means to force an entrepreneur to repay is to contract the entrepreneur’s asset (project) as collateral. If the entrepreneur does not repay, the bank can threaten to liquidate the entrepreneur’s project to sell in the secondary market at T_1 . We denote by P the market price of the asset (project) in the secondary market. If an entrepreneur has the full bargaining power in renegotiating with his bank, then an entrepreneur will never be able to creditably commit to repay more than P at T_1 (see Hart and Moore (1994)). Therefore, the collateral value of an entrepreneur’s asset at T_1 is P .⁸ Both P and r will be endogenized.

2.4 Government (policymaker)

After the economy suffers the systemic liquidity shock, the government chooses an amount of liquidity, Q , to inject into the banking system at T_0 , where $Q \in [0, \bar{Q}]$ and \bar{Q} is a constant reflecting the government’s constraint in economic stimulus. For simplicity and without loss of generality, we assume that each bank obtains a fixed amount of liquidity, aggregating to Q . All quantities in our model are in ‘real’ and not ‘nominal’ terms.

The injected liquidity Q in our model represents loanable funds. As in the literature on unconventional credit policies, we have abstracted away the institution and assumed that the government can directly determine the amount of loanable funds in the banking system. This simplification is to capture the fact that the government can use various policy tools to influence bank credit available to the economy, for example, the policies of direct lending to financial institutions, equity injections to increase bank capital, and so on as were observed in the recent crisis (Bernanke (2009)).

One interpretation of the liquidity injection Q is a fiscal stimulus, such as the credit expansions that many countries have conducted in response to economic crises. For example, the government borrows Q amount of *real goods* from an unmodelled source (e.g., foreign countries) to support the domestic economy, and the Q amount of borrowing is fully repaid later with the government’s revenue (derived from its lending to the domestic banks).

We will specify the objective function of the government later. Figure 1 summarizes the main setup of the model.

⁸We will show that $P > X$, so the long-term debt with maturity at T_2 is not optimal or feasible for some entrepreneurs since they can raise less external financing by using long-term debt than by using short-term debt. This is in the spirit of Hart and Moore (1998) on the optimal debt maturity choice. Also, if the support of B is assumed to be $[X, I]$, long-term debt becomes infeasible for all entrepreneurs.

the liquidity investment, denoted B^* , is

$$B^* = \frac{P}{1+r}. \quad (1)$$

We will verify, by considering their participation conditions, that entrepreneurs $B \in [0, B^*]$ undertake the liquidity investment while entrepreneurs $B \in (B^*, I]$ do not.

Second, the credit market must clear at T_0 . The supply of funds is from banks, which have two sources of funding: the liquidity injection Q and the deposits from the non-investing entrepreneurs. The total demand of funds is by the investing entrepreneurs. Thus, we have

$$\int_{B^*}^I (I-B)f(B)dB + Q = \int_0^{B^*} Bf(B)dB. \quad (2)$$

Adding $\int_0^{B^*} (I-B)f(B)dB$ to both sides of this equation, equation (2) can be equivalently rewritten as

$$\int_0^I (I-B)f(B)dB + Q = I \cdot F(B^*). \quad (2')$$

Third, solving for the market equilibrium of the secondary asset market at T_1 gives the equilibrium asset price P . Entrepreneurs have different beliefs at T_1 . The asset valuation under high beliefs, denoted $\mathbb{E}^H(\tilde{x})$, is $\mathbb{E}^H(\tilde{x}) = u \cdot \theta_H + d \cdot (1 - \theta_H)$. Likewise, the asset valuation under low beliefs, denoted $\mathbb{E}^L(\tilde{x})$, is $\mathbb{E}^L(\tilde{x}) = u \cdot \theta_L + d \cdot (1 - \theta_L)$. Clearly, $\mathbb{E}^H(\tilde{x}) > \mathbb{E}^L(\tilde{x})$. The difference in valuations among entrepreneurs motivates them to trade. Also, as in Shleifer and Vishny (1992), only industry participants with previous periods of experience can operate the assets to generate cash flows at T_2 . Thus, buyers in the secondary market are entrepreneurs with high beliefs. Buyers can not only use their own funds but also use leverage (i.e., margin) when buying.

Based on the above analysis, we have the asset price:

$$P = \begin{cases} \mathbb{E}^H(\tilde{x}) & \text{if } \Gamma(B^*, r) > \mathbb{E}^H(\tilde{x}) \\ \Gamma(B^*, r) & \text{if } \Gamma(B^*, r) \in [\mathbb{E}^L(\tilde{x}), \mathbb{E}^H(\tilde{x})], \\ \mathbb{E}^L(\tilde{x}) & \text{if } \Gamma(B^*, r) < \mathbb{E}^L(\tilde{x}) \end{cases} \quad (3)$$

where

$$\Gamma(B^*, r) = \frac{\pi \left\{ \int_0^{B^*} [C - B(1+r)] f(B) dB + \int_{B^*}^I (1+r)(I-B)f(B)dB \right\} + X}{1-\pi},$$

or $\Gamma(B^*, r) = \frac{\pi[C \cdot F(B^*) - Q(1+r)] + X}{1-\pi}$ by using (2).

In (3), the asset price reflects not only the asset's expected *future* fundamental value at T_2 but also the *current* liquidity that buyers can access at T_1 . The current liquidity at T_1 available to

buyers, which is the numerator of $\Gamma(B^*, r)$, has two components. First, an entrepreneur who made the liquidity investment needs to repay his bank loans resulting in his net liquidity of $C - B(1 + r)$; an entrepreneur who did not make the investment can withdraw his deposits from banks resulting in his net liquidity of $(1 + r)(I - B)$. As all the entrepreneurs have the experience in managing the assets before T_1 , buyers are of total measure π . Thus, the aggregate internal fund of buyers at T_1 is captured by the first term of the numerator of $\Gamma(B^*, r)$.¹⁰ Second, a buyer uses his own asset plus his *bought assets* as collateral for borrowing, and he can borrow an amount X against each asset.¹¹ Hence, the aggregate borrowing of buyers in the economy is X . The denominator of $\Gamma(B^*, r)$ is the quantity of assets put up for sale.

Moreover, the asset price is truncated by upper and lower bounds, reflecting its dependence on the asset's expected future *fundamental* value at T_2 . If the asset price calculated in Γ is higher than $\mathbb{E}^H(\tilde{x})$, then in equilibrium the asset price is $\mathbb{E}^H(\tilde{x})$, at which the entrepreneurs with high beliefs are indifferent between buying and not, and some of whom do not participate in buying. A similar argument applies to the other extreme.

In what follows, we denote the pricing function of (3) as $P = p(\tilde{x}, B^*, C, X, r)$.

Finally, we check entrepreneurs' participation condition at T_0 . Given the interest rate r , if an entrepreneur with internal capital A borrows an amount $B = I - A$ to invest in his project, his payoff is $C - B(1 + r)$. Alternatively, the entrepreneur can deposit his internal capital in banks and realize a payoff of $A(1 + r)$. Thus, the entrepreneur is willing to invest if and only if

$$C - B(1 + r) \geq A(1 + r) \quad \Leftrightarrow \quad C - I(1 + r) \geq 0. \quad (4)$$

For simplicity, we focus on the set of equilibria in which inequality (4) is satisfied, that is where all entrepreneurs are willing to invest. Proposition 1 follows.

Proposition 1 *The equilibrium of the one-sector economy is characterized by a triplet $\{B^*, P, r\}$, which, given Q , solves the system of equations (1) to (3), and satisfies condition (4).*

To summarize, the analysis above captures the endogenous feedback loop under liquidity injections between bank lending, the asset price, and the collateral value, illustrated in Figure 2. Our paper provides a new micro-foundation for the endogenous feedback loop.

¹⁰Condition (4) later implies that every investing entrepreneur has sufficient cash to repay his debt at T_1 . That is, there is no default.

¹¹We focus on the equilibrium that the sellers' funds are big enough to satisfy the buyers' borrowing as in Geanakoplos (2010) and Simsek (2013).

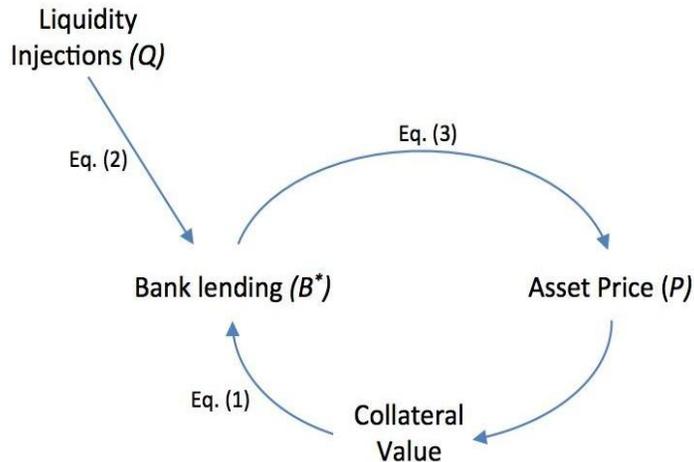


Figure 2: Feedback loop

3.2 Characterizing the equilibrium

First, we examine how liquidity injections impact the asset price P , i.e., the function $P(Q)$. Formally,

Lemma 1 *The equilibrium price Γ is increasing in Q if $C - I(1 + r) > 0$ in equilibrium.*

Proof. See Appendix. ■

Crucially, we need to consider the lower bound of P in (3). Proposition 2 follows.

Proposition 2 *If $X \leq \underline{X}$, where \underline{X} is a (positive) cutoff, the equilibrium asset price $P(Q)$ is a constant, equal to $\mathbb{E}^L(\tilde{x})$, no matter the size of the liquidity injection Q ($\in [0, \bar{Q}]$). If $X > \underline{X}$, $P(Q)$ is (weakly) increasing in Q under a sufficiently low π .¹²*

Proof. See Appendix. ■

Proposition 2 states that asset specificity X , which captures *financial friction for speculation*, plays an important role in determining the asset price. Proposition 2 gives the cleanest case for liquidity injections having little impact on the asset price.¹³ This happens when the financing friction is sufficiently high (X sufficiently low). If X is low, the buyers cannot use much leverage (i.e., margin financing). Hence, the asset price is low. It is possible that X is so low that the asset price is trapped at the lower bound $\mathbb{E}^L(\tilde{x})$ no matter what Q ($\in [0, \bar{Q}]$) is; in that case, P is not affected by Q .

¹²For our purpose, we focus on the set of equilibria in which Γ is lower than and not binding at $\mathbb{E}^H(\tilde{x})$. This can be achieved by assuming that θ_H and thus $\mathbb{E}^H(\tilde{x})$ are sufficiently big, *ceteris paribus*.

¹³For the benefit of a clean analysis and for our purpose, we have divided X into two regions in the analysis: $X \leq \underline{X}$ and $X > \underline{X}$. The merit of the cleanness can be further seen later when we discuss Figures 4a and 4b. The results of the paper however hold generally (for some \bar{Q}).

Next, we examine the equilibrium interest rate, i.e., the function $r(Q)$. Proposition 3 follows.

Proposition 3 *If the financing friction is high, $X \leq \underline{X}$, the equilibrium interest rate $r(Q)$ is strictly decreasing in Q ($\in [0, \bar{Q}]$). For lower financing friction, $X > \underline{X}$, under some distribution $f(B)$ and some parameter values, $r(Q)$ is U-shaped in Q , that is, there exists a minimum $r(Q)$, denoted r_{\min} , for an interior $Q \in (0, \bar{Q})$.*

Proof. See Appendix. ■

The interest rate equilibrates the *effective* demand for liquidity, captured by the collateral value (asset price) P , and the supply of liquidity, captured by the threshold B^* , through $1 + r = \frac{P(Q)}{B^*(Q)}$. Proposition 3 delineates two important cases. First, if the asset price is constant, then the interest rate certainly decreases in liquidity injections because the supply of loans increases with liquidity injections. Second, the asset price may increase very fast, faster than B^* , in which case the interest rate increases with liquidity injections. In fact, the latter case happens when the density $f(B)$ is thick in some region of B , by noting that the responsiveness of P (versus B^*) to the liquidity injection Q is crucially determined by the total additional funds generated by the liquidity investment, $C \cdot f(B^*)$. Figure 3 depicts the two cases.

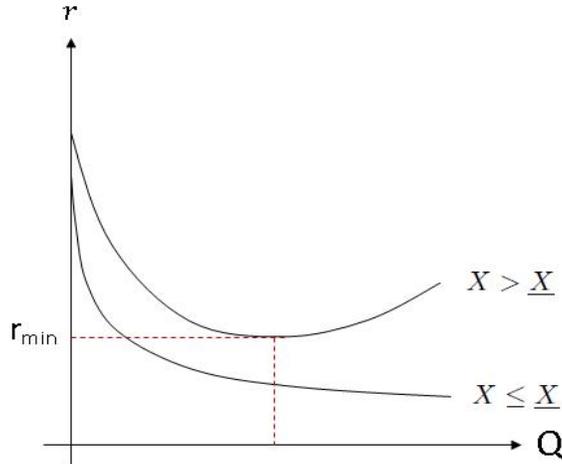


Figure 3: Equilibrium interest rate for the one-sector economy

Remark We use Figures 4a and 4b to explain the above mechanisms in the framework of the demand and supply equilibrium. The effective demand of liquidity is a decreasing function of the interest rate (for a given collateral value), i.e., the higher the interest rate, the lower the demand. Thus, the additional supply of liquidity typically causes the equilibrium interest rate to fall, which is the case of Figure 4a. However, in general equilibrium, the supply may also change the effective demand, i.e., the effective demand curve shifts upward because the collateral value increases. The demand curve may move upward very fast, and hence the equilibrium interest rate

is non-monotonic in liquidity supply, which is the case of Figure 4b. Formally, in our model, the aggregate supply of liquidity is $Q + \int_{B^*}^I (I - B)f(B)dB$ and the aggregate demand is $\int_0^{B^*} Bf(B)dB$. By (2'), we can equivalently rewrite the supply as $S(Q) = Q + \int_0^I (I - B)f(B)dB$ and the demand as $D(r; Q) = I \cdot F\left(\frac{P(Q)}{1+r}\right)$, where both supply and demand are functions of Q . In Figure 4a, $P(Q)$ is a constant while in Figure 4b, $P(Q)$ is increasing in Q .

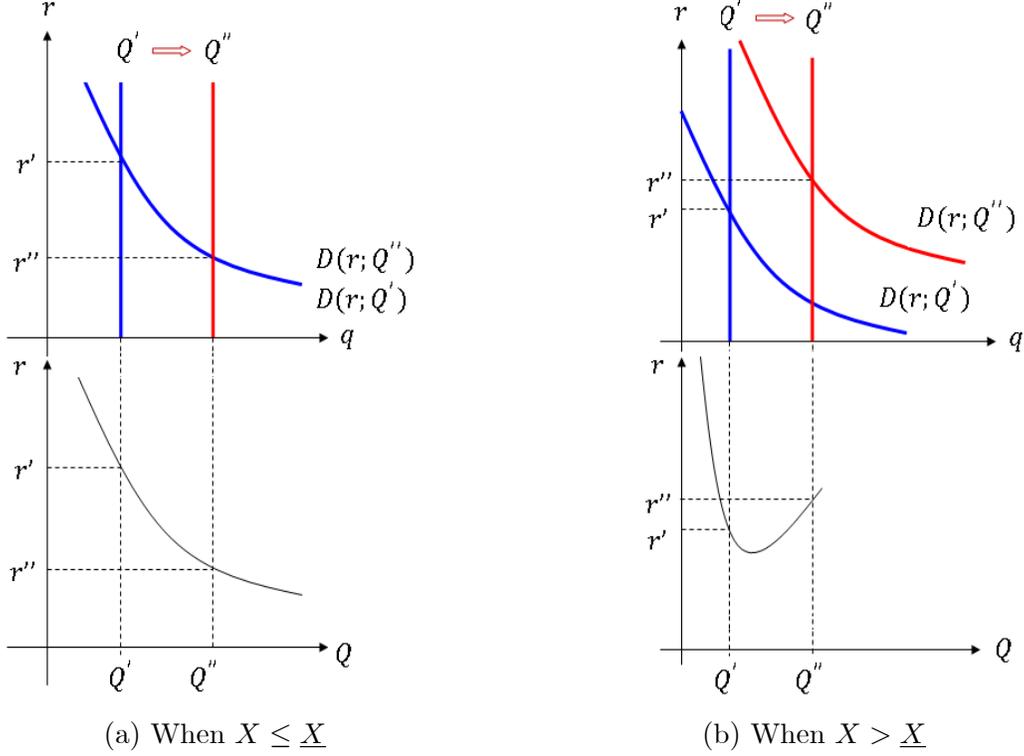


Figure 4: Liquidity market equilibrium

4 Two-sector economy

We now consider the equilibrium with two sectors, so that we can study the distribution of liquidity and the possible misallocation of credit, the goal of our study. The difference in financial friction for speculation across sectors, originating in asset specificity, results in heterogeneity in speculative activities across those sectors. This heterogeneity leads to asymmetric responses of collateral prices to liquidity injections across the two sectors. As the two sectors compete for loans (liquidity) in the same bank credit market, the crowding-out effect can occur.

4.1 Two-sector equilibrium: the crowding-out effect

In order to highlight the key mechanism underlying the crowding-out effect, at this stage we assume that the two sectors differ only in their asset specificity (the term X). Sector 1 has higher friction,

a lower X denoted X_1 , where $X_1 \leq \underline{X}$, while Sector 2 has lower friction, a higher X denoted X_2 , where $X_2 > \underline{X}$.¹⁴ Note that the (exogenous) X determines *asset leverage* in speculation at T_1 while the (endogenous) B^* measures *corporate leverage* in a sector at T_0 .

The government can decide the aggregate liquidity to inject into the economy but cannot control the allocation of the liquidity across the two sectors in the economy, which is determined by banks based on market forces. Market forces mean that: i) Banks make loans only based on firms' *repaying ability* or collateral values (no matter which sector entrepreneurs are from); ii) There is a single interest rate in the economy (for both sectors). Also, the government cannot control the (real) interest rate directly.

We solve for the two-sector economy equilibrium: for a given Q , how the liquidity is allocated and distributed across the two sectors. The equilibrium is given by the following equation system and condition:

$$r = \frac{P_i}{B_i^*} - 1 \quad (5a)$$

$$\int_{B_i^*}^I (I - B) f(B) dB + Q_i = \int_0^{B_i^*} B f(B) dB \quad (5b)$$

$$P_i = p(\tilde{x}, B_i^*, C, X_i, r) \quad (5c)$$

$$C - I(1 + r) \geq 0 \quad (5d)$$

$$Q = \sum_i Q_i, \quad (5e)$$

where $i = 1$ and 2 .

In (5a)-(5e), the variables with superscript i denote the variables for sector i , where $i = 1$ and 2 . In particular, Q_i is defined as the *net* amount of outside-sector liquidity entering sector i , or the *net* liquidity inflow into sector i . Equations (5a), (5b) and (5c), and condition (5d) correspond, in order, to the respective equations (1), (2) and (3), and condition (4) of the one-sector economy. The three equations ((5a) through (5c)) and condition (5d) give the equilibrium *within* each sector. The interaction *between* the two sectors is given by (5a) and (5e). First, equation (5a) states that there is a single interest rate, in equilibrium, for the two sectors. Second, equation (5e) reflects that the aggregate outside-sector liquidity of the two sectors must equal Q .

In the two-sector economy, there is an integrated bank credit market at T_0 , through which capital can flow across sectors. That is, a non-investing entrepreneur in one sector saves his spare internal

¹⁴For cleanness, we consider that $X_2 > \underline{X} \geq X_1$. When $X_2 > X_1 \geq \underline{X}$, the result in this section (i.e., the unique optimal level of liquidity injection as will be shown in Propositions 5, 7 and 8) still holds. However, a stricter condition is required, that is, the maximum amount of allowed liquidity injection \bar{Q} cannot be too large. The reason is that when $X_2 > X_1 > \underline{X}$, the interest rates in both sectors might be increasing in the liquidity inflow when the liquidity inflow is sufficiently high. Hence, the crowding-out effect occurs only when \bar{Q} is not very large.

capital in banks and can actually become an ultimate liquidity provider to investing entrepreneurs in the other sector. In fact, in (5a)-(5e), even when $Q = 0$, Q_i might not be equal to zero, in which case, it measures the liquidity flow across the two sectors. In contrast, the secondary asset markets for Sectors 1 and 2 at T_1 are segmented. That is, only industrial participants active in a particular sector can buy assets from their peers in the same sector. We will relax the assumption of asset market segmentation in Section 5.

Under certain parameter conditions, the equation system and condition of (5a)-(5e) have a unique solution and hence there is a unique equilibrium. We can think that there are two steps in solving for the two-sector equilibrium in (5a)-(5e). First, given Q_i for each sector, solve for the equilibrium within each sector, that is, solve for the triplet $\{B_i^*, P_i, r_i\}$. In particular, we obtain the function $r_i(Q_i)$. Second, by considering the link between the two sectors, (5a) and (5e), we can work out Q_i (for $i = 1$ and 2) for a given Q . That is, by considering $r_1(Q_1) = r_2(Q_2) = r$ and $Q_1 + Q_2 = Q$, we obtain the unique Q_1 and Q_2 , and r . We have Proposition 4.

Proposition 4 *The (market) equilibrium of the two-sector economy is characterized by a set $\{B_i^*, P_i, Q_i, r\}$ where $i = 1$ and 2 , which, given Q , solves the system of equations (5a), (5b), (5c) and (5e), and satisfies condition (5d). If $r_1(Q_1)$ is strictly decreasing in Q_1 and $r_2(Q_2)$ is \cup -shaped in Q_2 , there is a unique equilibrium for the two-sector economy.*

Proof. See Appendix. ■

We conduct the comparative static analysis on $Q_1(Q)$, and find a unique Q that maximizes Q_1 .

Proposition 5 *If $r_1(Q_1)$ is strictly decreasing in Q_1 and $r_2(Q_2)$ is \cup -shaped in Q_2 , there is a unique Q , denoted by Q^* , that maximizes Q_1 (or B_1^*).*

Proof. See Appendix. ■

Figure 5 shows the intuition for determining the unique Q^* . In the figure, $Q^* = Q_1 + Q_2$. If the liquidity injection exceeds this level, any additional liquidity flows to Sector 2 and, further, some liquidity in Sector 1 is actually *squeezed out* due to the increased interest rate. Therefore, overall, the liquidity in Sector 2 increases while that in Sector 1 decreases. In fact, if Q increases above Q^* , Sector 1 cannot attract more liquidity as there is otherwise no interest rate at which the credit market across the two sectors can clear; thus Sector 2 must attract more liquidity, which will certainly increase the interest rate as we move along the increasing part of the liquidity curve of Sector 2.

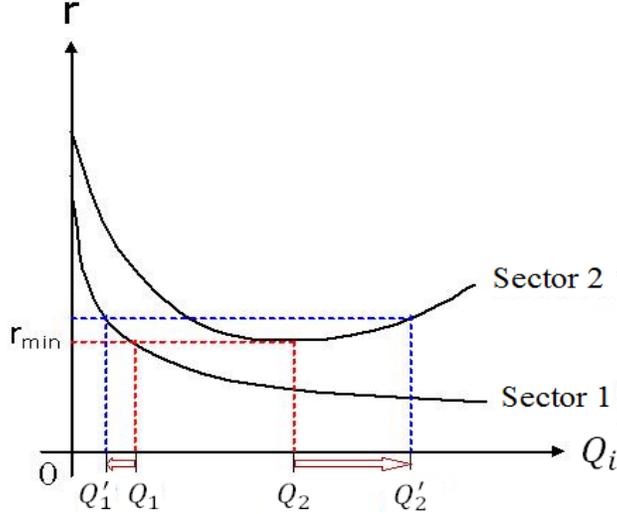


Figure 5: Crowding-out effect in the two-sector economy equilibrium

It is important to examine the process by which the new equilibrium is reached when some additional amount of liquidity (beyond Q^*) is injected. In Figure 5, in which an additional amount of liquidity $\Delta Q = (Q'_1 + Q'_2) - (Q_1 + Q_2)$ is injected, we examine how Q_1 reaches Q'_1 . Suppose initially all additional liquidity, ΔQ , enters Sector 2; this would push up the interest rate; the higher interest rate would *squeeze* some liquidity out of Sector 1, which means that more liquidity would flow into Sector 2, pushing up the interest rate further, and so on in a spiral. Essentially, there is a spiral in reaching the new equilibrium. From the perspective of Sector 2, the spiral results in a multiplier $\frac{Q'_2 - Q_2}{\Delta Q} > 1$; that is, although the (aggregate) additional amount of liquidity injection is ΔQ , the increment of liquidity in Sector 2 is actually more than ΔQ ; the leap is at the expense of liquidity flows to Sector 1, causing a liquidity drain out of Sector 1. In short, one sector enters a liquidity-asset price ‘inflationary’ cycle while the other enters a ‘deflationary’ cycle, and the two cycles reinforce each other.

In sum, we can divide the support of liquidity injection, Q , into two regions: $Q \leq Q^*$ and $Q > Q^*$. Proposition 6 follows.

Proposition 6 *When $Q \leq Q^*$, there is an ‘allocation’ effect, i.e., liquidity in both sectors increases with liquidity injections but Sector 1 obtains less liquidity than Sector 2. When $Q > Q^*$, there is a ‘crowding-out’ effect, i.e., more liquidity injected increases the liquidity in Sector 2 but reduces it in Sector 1; the crowding-out occurs in a self-reinforcing spiral.*

Proof. *See Appendix.* ■

Although the liquidity investments in the two sectors have the same surplus (i.e., $C - I$), the liquidity is unevenly distributed across the two sectors. In particular, if too much liquidity is

injected into the economy, Sector 1 can actually suffer a liquidity outflow (deficit). As long as the collateral value (the asset price) in one sector increases faster than that in the other sector, the potential for crowding-out exists.

4.2 Government decision and credit misallocation

Now we examine the decision of the government. To make the problem interesting and relevant, we assume that the two sectors differ in both their cash flow at T_1 and their asset specificity while all other aspects are the same. Specifically, we assume that $C_1 > C_2$ and $X_1 < X_2$, where C_i and X_i are, respectively, the cash flow at T_1 and the asset specificity for sector i , and $i = 1$ and 2 . That is, Sector 1 has a higher cash flow and higher asset specificity.

Parallel to (5a)-(5e), the market equilibrium of the two-sector economy for a fixed Q is given by

$$r = \frac{P_i}{B_i^*} - 1 \tag{6a}$$

$$\int_{B_i^*}^I (I - B) f(B) dB + Q_i = \int_0^{B_i^*} B f(B) dB \tag{6b}$$

$$P_i = p(\tilde{x}, B_i^*, C_i, X_i, r) \tag{6c}$$

$$C_i - I(1 + r) \geq 0 \tag{6d}$$

$$Q = \sum_i Q_i, \tag{6e}$$

where $i = 1$ and 2 , and both X_i and C_i are different across sectors. Similar to Proposition 4, we can show that under general conditions the equilibrium given by (6a)-(6e) exists and is unique.

We model two alternative objective functions of the government, which deliver qualitatively equivalent results. That is, there is an optimal level of liquidity injection for the government and the liquidity injection should not be too large. The first objective function is that the government targets to maximize the number of projects in Sector 1 that can undertake the liquidity investment. This is because $C_1 > C_2$, which means that projects in Sector 1 should receive liquidity injection from the efficiency point of view. This may also be because the government cares more about outcomes such as employment (e.g., small and medium-sized businesses) than purely firms' profits. If Sector 1, relative to Sector 2, is disproportionately more important in these aspects, the government may make Sector 1 its first priority in economic stimulus.¹⁵ The government's optimization

¹⁵In terms of modelling, we can assume that the liquidity investment for projects in Sector 1 generates not only cash flow C but also some non-pecuniary payoffs.

problem is:

$$\begin{aligned} \max_Q Q_1 & \qquad \qquad \qquad \text{(Program 1)} \\ \text{s.t.} \quad & (6a) - (6e). \end{aligned}$$

In program 1, maximizing Q_1 is equivalent to maximizing B_1^* . The government chooses a Q to maximize the objective function subject to the two-sector market equilibrium given by (6a)-(6e). Similar to Propositions 5 and 6, we have Proposition 7.

Proposition 7 *Under certain conditions, Program 1 has a unique interior optimal $Q \in (0, \bar{Q})$, denoted by Q^* . The properties in Proposition 6 carry over.*

Proof. See Appendix. ■

Proposition 7 implies that excessive liquidity injections lead to a *misallocation of liquidity* in the economy: the sector with lower-surplus projects (i.e., $C_2 - I$) obtains more liquidity while the sector with higher-surplus projects (i.e., $C_1 - I$) *loses* liquidity. In fact, the optimal resource allocation requires the marginal product (return) to be equalized across sectors (Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)). In our model, the marginal return of investment for sector i is a constant $\frac{C_i}{I}$, and Sector 1 has a higher marginal return (i.e., $\frac{C_1}{I} > \frac{C_2}{I}$). Hence liquidity should be allocated toward Sector 1.

It is helpful to examine the return to capital in the aggregate economy, denoted by \bar{R} , which is given by

$$\bar{R} = \frac{C_1 \cdot F(B_1^*) + C_2 \cdot F(B_2^*)}{I \cdot F(B_1^*) + I \cdot F(B_2^*)}.$$

It is easy to show

$$\bar{R} = R_1 \omega + R_2 (1 - \omega) \tag{7}$$

where $R_i \equiv \frac{C_i}{I}$ is the capital return for sector $i = 1$ and 2, and $\omega = \frac{F(B_1^*)}{F(B_1^*) + F(B_2^*)}$, the share of Sector 1 in terms of liquidity investment. That is, the return to capital in the aggregate economy is the weighted average of the returns to capital across the two sectors, weighted by their shares in liquidity investment. Clearly, ω is decreasing in Q for $Q > Q^*$ due to the crowding-out effect. Corollary 1 follows.

Corollary 1 *The return to capital of the aggregate economy, \bar{R} , is decreasing in Q for $Q \geq Q^*$.*

Corollary 1 shows that despite a constant return to capital in each individual sector, the aggregate economy exhibits a decreasing return to capital when $Q > Q^*$. This is because more liquidity injection results in a more severe misallocation of credit across sectors when $Q > Q^*$.

The second objective function is that the government is to maximize the aggregate surplus of liquidity investments in the economy. The aggregate surplus is given by

$$W = (C_1 - I) \cdot F(B_1^*) + (C_2 - I) \cdot F(B_2^*).$$

By using (2'), it is easy to show that W can be rewritten as

$$W = C_1 \cdot F(B_1^*) + C_2 \cdot F(B_2^*) - 2 \int_0^I (I - B)f(B)dB - Q,$$

where the first two terms are the aggregate cash income at T_1 in the economy, the third term is the funding cost of the internal funds of entrepreneurs, and the fourth term is the funding cost of the government's liquidity injection. The aggregate surplus W is, in the end, divided among investing entrepreneurs (profits), non-investing entrepreneurs (interest on deposits) and the government (interest on Q) (see the proof of Proposition 8).

Remark In general, it is difficult to conduct welfare analysis on models with heterogeneous beliefs (see, e.g., Brunnermeier, Simsek and Xiong (2014)). However, this is not the case with our model. In our model, we assume that the cash flow \tilde{x} at T_2 (over which firms develop heterogeneous beliefs) and the distribution of beliefs (i.e., the probability π) are identical across the two sectors. More importantly, the asset, which generates cash flow \tilde{x} , is *in place* at T_0 ; liquidity injections do not affect \tilde{x} . Liquidity injections only impact the liquidity investment I and the cash flow C subsequently. As a result, we can conduct the welfare analysis by calculating the incremental income from the liquidity injection minus by the funding cost.

Under the alternative objective function, the government's optimization problem is:

$$\begin{aligned} \max_Q W & \qquad \qquad \qquad \text{(Program 2)} \\ \text{s.t.} & \quad (6a) - (6e). \end{aligned}$$

Proposition 8 follows.

Proposition 8 *Under certain conditions, Program 2 has a unique interior optimal $Q \in (0, \bar{Q})$. That is, the government has a unique optimal level of liquidity injection to maximize the aggregate surplus of liquidity investments in the economy.*

Proof. See Appendix. ■

The intuition behind Proposition 8 is as follows. We can decompose the aggregate income of

the economy at T_1 as

$$C_1 \cdot F(B_1^*) + C_2 \cdot F(B_2^*) = [C_1 \cdot F(B_1^*(Q_1^*)) + C_2 \cdot F(B_2^*(Q_2^*))]$$

$$+ \left[\underbrace{C_2 \cdot \begin{pmatrix} F(B_1^*(Q_1)) + F(B_2^*(Q_2)) \\ -F(B_1^*(Q_1^*)) - F(B_2^*(Q_2^*)) \end{pmatrix}}_{\text{additional injected liquidity entering Sector 2}} - \underbrace{(C_1 - C_2) \cdot (F(B_1^*(Q_1^*)) - F(B_1^*(Q_1)))}_{\text{some liquidity moving out of Sector 1 to Sector 2}} \right],$$

where Q_i^* is the net liquidity inflow for Sector i when $Q = Q^*$ in Proposition 7. An increase in Q beyond Q^* has two opposite forces on income at T_1 : additional injected liquidity entering Sector 2 increases income, but more injected liquidity also squeezes more liquidity out from Sector 1 and into Sector 2, decreasing income because $C_1 > C_2$. Under certain conditions, the second force dominates the first force when Q is too high, and, therefore, there exists an optimal Q .

5 Model extension

In this section, we study several extensions of the model.

5.1 Segmentation of asset markets

In the main model, we assumed that the secondary asset markets for Sectors 1 and 2 at T_1 are segmented. We can alternatively assume that the secondary asset markets are not completely segmented. Specifically, we can assume that entrepreneurs from Sector 1 (e.g., the real sector) can trade in both asset markets, while entrepreneurs from Sector 2 (e.g., the real estate or financial sector) can only trade in asset market 2. We show that under this alternative assumption our model results do not change qualitatively.

Intuitively, under the alternative assumption, Sector 2 will end up with more speculators while Sector 1 will lose some of its speculators to Sector 2. As a result, the asset price in Sector 2 is boosted further while that in Sector 1 remains trapped at the lower bound. In particular, the asset price in Sector 2 is still increasing in the liquidity injection (that is, Propositions 2 and 3 still hold).

Formally, to examine the impact of imperfect asset market segmentation, we explore three alternative scenarios regarding agents' beliefs: 1) Beliefs of an entrepreneur are perfectly correlated in the two asset markets; 2) Beliefs of an entrepreneur are not perfectly correlated in the two asset markets; 3) There exist no heterogeneous beliefs among entrepreneurs and thus no speculative trade in Sector 1 while there does in Sector 2. We show that if the secondary asset markets are not completely segmented, Propositions 2 and 3 hold (see the appendix).

5.2 Alternative borrowing constraints

The borrowing-constraint setup in our model is consistent with the one in Kiyotaki and Moore (1997), where the borrowing in the *current* period depends on the asset price in the *next* period. This is also true in our model. In fact, we can rewrite the asset price in (3) as $\Gamma \cdot (1 - \pi) = e + b$ with $b \leq X \cdot (1 - \pi)$, where $e = \pi [C \cdot F(B^*) - Q(1 + r)] + \pi X$. The borrowing at T_1 (the term b) depends on the “asset price” at T_2 (the term X). Note that because our model is a finite-horizon model, the asset price at the *final* date T_2 is equal to the *collectable* or *contractible* part of “dividends” at that date, which is the asset specificity term X .

Under the alternative assumption that the borrowing in the *current* period depends on the asset price in the *current* period, our model results do not change qualitatively. In this case, Γ in (3) would be written as $\Gamma \cdot (1 - \pi) = e_c + (\Gamma\lambda)\pi + (\Gamma\lambda) \cdot (1 - \pi)$ where $e_c = \pi [C \cdot F(B^*) - Q(1 + r)]$ (which is increasing in Q by Lemma 1) and λ is the margin leverage ratio. Note that the borrowing against both own existing assets and bought assets depends on the asset price in the current period (i.e., the borrowing amount $\Gamma\lambda$ per unit asset collateral depends on the current asset price Γ). So $\Gamma = \frac{e_c}{1 - \pi - \lambda}$, which implies $\frac{\partial \Gamma}{\partial \lambda} > 0$ and $\frac{\partial^2 \Gamma}{\partial \lambda \partial Q} > 0$, meaning that the higher the λ , the more sensitive the response of Γ to liquidity injection Q . Hence, Propositions 2 and 3 still hold.

5.3 Different interest rates across sectors

Within the model, there is no reason for banks to charge different interest rates (between T_0 and T_1) to entrepreneurs from different sectors. This is because there is no default of bank loans in equilibrium in our model, and thus *no differential riskiness* of bank loans across sectors. In other words, the interest rate on bank loans in our model is in fact the real *risk-free* interest rate, which of course is a single one in the economy.

If we consider the real world *outside* the model, the frictions that our model does not study and abstracts away, such as adverse selection, might lead to different *risky* interest rates across sectors due to different risk premia across sectors. Nevertheless, even if we relax our original assumption by allowing for differences in interest rates across the two sectors but with some extent of co-movement, our model results do not change qualitatively. Specifically, we can assume that $r_1 - r_2 = \Delta r$, where r_i is the interest rate on bank loans (between T_0 and T_1) in sector $i = 1$ and 2, and Δr is a constant positive spread; that is, there is a constant spread in interest rates on bank loans between the two sectors (for exogenous reasons). We prove that our model results do not change qualitatively under the new assumption.

5.4 Alternative sources of heterogeneity across sectors

Some other dimensions of heterogeneity documented empirically are *endogenous* in our model. For example, heterogeneity in *corporate leverage*, *recovery rate*, *secondary market trading* across sectors is endogenous in our model. While alternative sources of heterogeneity across sectors might also generate the misallocation of liquidity, formalizing clearly the mechanism underlying it may not be straightforward. In particular, our model shows the effect of two *ranges* of liquidity injection Q : When $Q \leq Q^*$, there is an ‘allocation’ effect; when $Q > Q^*$, there is a ‘crowding-out’ effect.

5.5 Crowding-out effect with a falling interest rate

One implication of the main model is that in equilibrium the crowding-out effect is accompanied by an increase in the interest rate. However, our model framework also shows that the crowding-out effect can be accompanied by a decrease in the interest rate. In fact, when $r_1(Q_1)$ is increasing in Q_1 (in some region of Q_1) and $r_2(Q_2)$ is decreasing in Q_2 (in some region of Q_2), such an equilibrium outcome can occur.

Figure 6 illustrates such an equilibrium. In the figure, when an additional $\Delta Q = (Q'_1 + Q'_2) - (Q_1 + Q_2)$ of liquidity is injected, Sector 2 enters a liquidity-asset price ‘inflationary’ cycle while Sector 1 enters a ‘deflationary’ cycle, and the two cycles reinforce each other. Intuitively, when Sector 1 loses some liquidity, its asset price will quickly fall, causing its affordable interest rate to go down. Because of the higher price (interest rate) elasticity of demand to liquidity for Sector 2, at the lower interest rate Sector 2 is able to absorb and accommodate more liquidity than the initial lost amount of Sector 1 and thus will necessarily suck additional liquidity out of Sector 1. Technical details are relegated to the appendix.

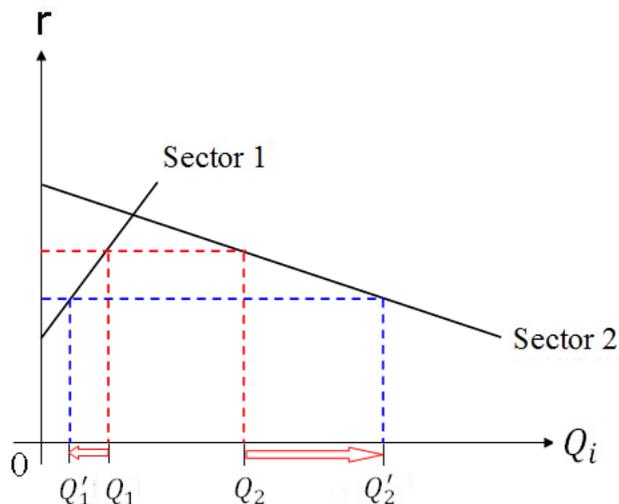


Figure 6: Crowding-out effect accompanied by a falling interest rate

6 Empirical evidence

In this section, we discuss empirical evidence in support of our model.

6.1 Evidence on asset specificity

A key mechanism of our model is to link asset specificity, secondary market trading, and corporate leverage. In our model, the cash flow X — the recovery part of cash flow in the event of default — measures asset specificity. Our model assumes asset specificity heterogeneity across sectors, and implies: i) The lower the asset specificity, the higher the recovery rate; ii) The lower the asset specificity, the higher the trading amount in the asset secondary market;¹⁶ iii) The lower the asset specificity, the higher corporate leverage. All these are strongly supported by empirical facts (see, e.g., Kim and Kung (2017)).

The following table, based on the data in Kim and Kung (2017), shows asset redeployability across industries, where asset redeployability is the measure of asset specificity with consensus in the literature. Lower redeployability means higher asset specificity.

Panel A: Descriptive statistics

	N Industries	Mean	SD	Min	Q1	Median	Q3	Max
Redeployability	9	0.222	0.061	0.109	0.185	0.221	0.274	0.293

Panel B: Most and least redeployable industries based on one-digit SIC

Redeployability	Most redeployable		Least redeployable	
	industry	Redeployability	Industry	Redeployability
0.293	Financial	0.109	Mining	
0.277	Wholesale Trade	0.175	Transportation	
0.274	Construction	0.185	Agriculture	
0.262	Retail Trade	0.201	Manufacturing	
0.221	Services			

Table 1: Measure of asset redeployability by Industries (Source: Kim and Kung (2017))

From table 1, asset redeployability varies significantly across industries. Notably, industries such as financial services, construction, and wholesale and retail trade have higher asset redeployability (i.e., lower asset specificity) while manufacturing industries have lower asset redeployability.

6.2 Shadow banking activities in China

While the formal banking system in China is tightly regulated (for example, regulation of interest rates on deposits and credit quota in commercial banks), the last decade has witnessed an explosive

¹⁶Based on Eq. (3) in our model, lower asset specificity (a higher X) corresponds to higher asset sales (the numerator of Γ), which is confirmed by Kim and Kung (2017) (see Panel B of Table 8 in their paper).

growth of unregulated shadow banking activities in China.¹⁷ According to a Moody’s report (2013), by the end of 2012, the total value of shadow banking products in China was 39% of GDP.

An important part of China’s shadow banking activities is the re-lending business (Du, Li and Wang (2016)) and broadly entrusted loans (Allen et al. (2015), Chen, Ren and Zha (2016)). The re-lending activity consists of non-financial firms with good access to formal finance acting as de facto financial intermediaries, i.e., borrowing from banks and lending to credit-constrained firms. In fact, such re-lending activities grew to large scale in Japan in the late 1980s and were an important force behind the boom-bust cycle of the Japanese economy (see Hattori, Shin and Takahashi (2009) for detailed evidence). The re-lending business in China is a natural outcome of financial repression whereby large privileged enterprises have access to formal finance with favorable conditions but small and medium-sized enterprises can hardly access formal finance (Du, Li and Wang (2016)). In examining the re-lending business in China, Du, Li and Wang (2016) document empirical evidence to answer two broad questions: who lends and who borrows. They find that: 1) State-controlled companies that have better access to financial markets are particularly active in re-lending; 2) Real-estate and construction firms, and small and medium-sized businesses (SMBs) are the main borrowers.

The findings of Du, Li and Wang (2016) and others on China’s shadow banking generally support the mechanism of our model. That is, the lenders in the shadow banking sector — the privileged enterprises (notably large central state-owned enterprises) that have access to formal finance with favorable conditions — correspond to “banks” in our model, while the two main borrowers — SMBs and the real estate sector — correspond to Sector 1 and Sector 2, respectively, in our model. When China implemented its credit expansion, a large volume of credit went to privileged enterprises that de facto acted as financial intermediaries and conducted re-lending. As the total volume of re-lending loans increased, it contributed to the overheating of one sector (i.e., the real-estate and construction sector) while liquidity in the other sector (i.e., SMBs) was crowded out through the channel of rising real interest rates. In fact, after China conducted its credit expansion in 2008-2009, one pronounced phenomenon was the surge in real estate prices, with a 50% increase within one year in many cities.¹⁸ Asset prices were also climbing in other asset classes, like commodities. In contrast, SMBs in China had an even harder time obtaining corporate liquidity following the economic stimulus.¹⁹ The underground interest rate reached 30% in some regions.²⁰ The People’s Daily wrote: “Massive funds pulled out the real sector and flowed into the real estate sector,

¹⁷See Allen et al. (2015), Elliott et al. (2015), Acharya et al. (2016), Chen, Ren and Zha (2016), Chen, He and Liu (2016), Du, Li and Wang (2016), Hachem and Song (2016), and Wang et al. (2016).

¹⁸See Fang, Gu, Xiong and Zhou (2015).

¹⁹See, e.g., the Citibank-CCER report of “Financing and growth of small and medium-sized businesses in China” (available in Chinese only at <http://www.ccstock.cn/stock/jinrongjigou/2011-07-21/A516377.html>).

²⁰<http://finance.ifeng.com/bank/pjrz/20100813/2507661.shtml> (the article is only available in Chinese). There are plenty of news reports on this on Chinese websites.

crowding out the real economy.”²¹ Chen, Liu, Xiong and Zhou (2017) provide evidence that the crowding-out effect led to substantial capital misallocation.²²

6.3 Misallocation of capital flows in EU countries

Reis (2013) argues that financial integration without the necessary financial deepening in some of the EU’s peripheral countries since the late 1990s led to massive foreign credit, mainly through domestic banks (serving as intermediaries), pouring into relatively unproductive firms in the nontradables sector at the expense of more productive tradables firms.²³ The misallocation of abundant capital flows from abroad explains the poor economic performance of these countries even before the global financial crisis of 2007-2009. Reis (2013) documents that in the period of 2000-2006 the shares of the tradables sectors (such as manufacturing) declined in terms of employment and nominal value added in the Portuguese economy while the shares of the nontradables sectors (such as real estate, wholesale and retail trade, community services) increased. Ireland and Spain had similar features. The share of the construction sector in Spain’s economy also rose significantly.

The misallocation of capital flows documented by Reis (2013) might be accounted for in our framework. From Table 1, the industries of financial services, construction, and wholesale and retail trade exhibit lower asset specificity; they are also coincidentally the less tradable industries in the framework of Reis (2013). In contrast, the industries of manufacturing and transportation have higher asset specificity and are also among the more tradable industries. Based on our model (see Proposition 6), when the capital flow to an economy is not large, there is an ‘allocation’ effect: liquidity in both sectors increases but the sector with higher friction obtains less liquidity than the sector with lower friction, and the real interest rate drops. The ‘allocation’ effect implies that the sector with lower friction will grow faster than the sector with higher friction, and hence its share in the economy will expand.²⁴

6.4 Japan’s experience in the 1980s

One would expect a negative relation between liquidity supply and interest rates: a lower supply of liquidity should increase interest rates. The Japanese experience during the 1980s, however, is a stark example of a tightening policy accompanying a (slight) decrease in real interest rates. In fact, the tightening policy in Japan in the second half of 1980s was followed by a fall in asset prices

²¹<http://www.chinanews.com/estate/2011/12-09/3519315.shtml> (the article is only available in Chinese).

²²See also Cong et al. (2017) and Huang et al. (2017).

²³See also Benigno and Fornaro (2014), Giavazzi and Spaventa (2011), and Gopinath et al. (2017). Reinhart and Kaminsky (1999) provide historical evidence on capital inflows, bank credit, and crises.

²⁴In our model, the crowding-out effect under $Q \geq Q^*$ is a sufficient but not necessary condition for a decreasing return to capital of the aggregate economy (see Corollary 1). In other words, when $Q < Q^*$, the ‘allocation’ effect may also cause a decreasing return to capital of the aggregate economy.

and thus a reduction in the collateral values of firm assets; the reduction in the creditworthiness of Japanese corporations at least in part contributed to lower demand for credit, driving down interest rates. Bernanke and Gertler (1995) write: “the crash of Japanese land and equity values in the latter 1980s was the result (at least in part) of monetary tightening; ... [T]his collapse in asset values reduced the creditworthiness of many Japanese corporations and banks...”. What happened recently in China can be regarded as the same sort of problem the Japanese faced, but in the opposite direction. That is, the massive liquidity injections and credit expansion in China created overheating in some sectors (e.g., the real estate sector), which led to the effective demand for credit shooting up, in turn causing real interest rates to rise.

7 Concluding remarks

This paper proposes a new channel for credit misallocation that links credit misallocation to credit expansion (or capital inflows). The study suggests that unconventional credit policy can have limited effectiveness in stimulating economic growth. While financial intermediaries have an (informational) advantage/expertise in allocating capital to firms, market frictions also mean that implementation of credit policy through financial intermediaries is imperfect. The paper implies that credit policy in conjunction with fiscal policy to target some specific sectors/industries may have better effects in economic stimulus. Regulation on the leverage level in more speculative industries when other industries are in distress may help reduce the crowding-out effect.

Appendix

A Numerical Example

We provide a numerical example for the crowding-out effect, illustrating the existence of relevant parameters in Propositions 2-6. That is, the numerical example is to highlight the qualitative (rather than quantitative) aspect of the model. The numerical examples for Propositions 7-8 will be provided in their proofs.

We choose parameter values as simple as possible. We set the parameter values for the project as $I = 1$, $C = 2.2$, $\pi = 0.4$, $\mathbb{E}^H(\tilde{x}) = 1.3668$, $\mathbb{E}^L(\tilde{x}) = 1.0334$. Note that we do not need to specify exactly u , d , θ_H and θ_L , any combination of which that satisfies that $u > d > 0$, $0 \leq X_1 < X_2 \leq d$, $u \cdot \theta_H + d \cdot (1 - \theta_H) = 1.3668$ and $u \cdot \theta_L + d \cdot (1 - \theta_L) = 1.0334$ works. The distribution of B is $f(B) = \log \frac{1}{1-B}$ for $B \in [0, 1)$. As the maximum total amount of liquidity that any one sector can demand is $Q^{\max} = \int_0^1 B f(B) dB = 0.75$, we set $\bar{Q} = 2Q^{\max} = 1.5$.

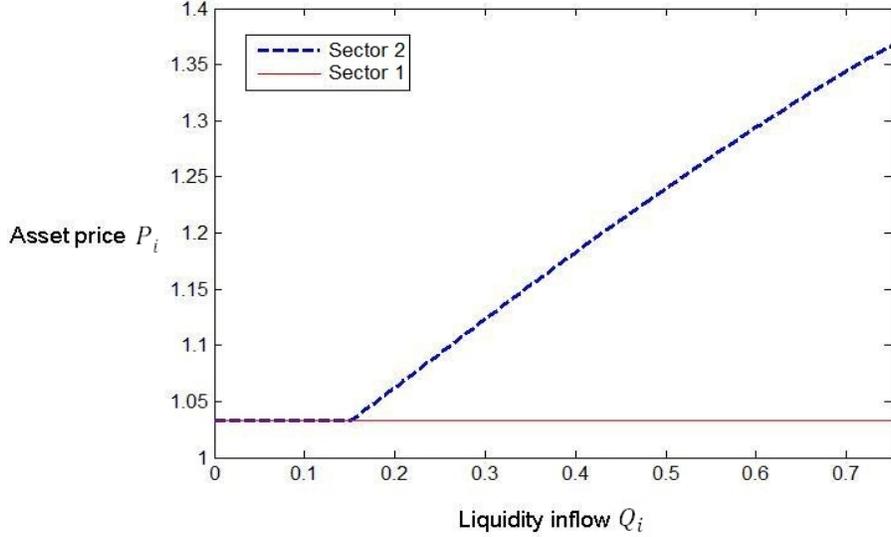
Given that $\mathbb{E}^L(\tilde{x}) = 1.0334$, we can calculate the threshold \underline{X} in Proposition 2, which is $\underline{X} = 0.0501$. We set $X_1 = 0.05$ and $X_2 = 0.35$, where $X_1 < \underline{X} < X_2$.

For Sector 1, we can work out that the asset price is $P_1 = 1.0334$ for any $Q_1 \in [0, 0.75]$. The asset price is trapped at $\mathbb{E}^L(\tilde{x}) = 1.0334$. The interest rate $r_1(Q_1)$ is a strictly decreasing function of liquidity inflow Q_1 .

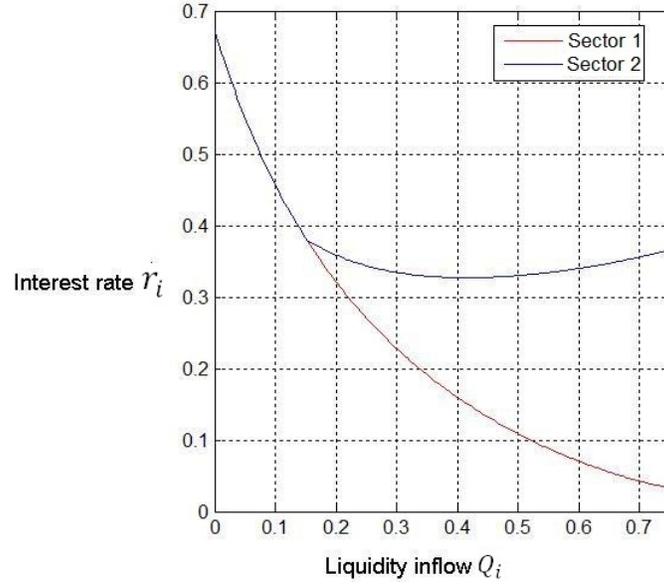
For Sector 2, the asset price $P_2(Q_2)$ is (weakly) increasing in Q_2 . When Q_2 is small, liquidity injections are not sufficient to push the asset price above $\mathbb{E}^L(\tilde{x})$ and the asset price is $P_2 = 1.0334$; when Q_2 is big enough, $P_2(Q_2)$ is strictly increasing in Q_2 , with the maximum asset price being $P_2(Q_2 = 0.75) = 1.3667$ (which is lower than $\mathbb{E}^H(\tilde{x}) = 1.3668$). As for the equilibrium interest rate $r_2(Q_2)$, it is non-monotonic and ‘U’-shaped, with the minimum interest rate being $r_{\min} = 0.3270$.

The optimal amount of liquidity injection to maximize liquidity investments in Sector 1 is $Q^* = 0.6113$, at which the distribution of the liquidity injection across the two sectors is $Q_1^* = 0.1950$ and $Q_2^* = 0.4163$, respectively.

Figure A1(a) shows the asset price response in each sector to its (net) liquidity inflow Q_i and Figure A1(b) depicts the interest rate response in each sector.



(a) Asset prices responses to liquidity inflow



(b) Interest rate responses to liquidity inflow

Figure A1: A numerical example

B Proofs

Proof of Lemma 1: By using the credit market clearing condition (2), the expression of $\Gamma(B^*, r)$ in (3) can be rewritten as

$$\Gamma(B^*, r) = \frac{\pi [C \cdot F(B^*) - Q(1+r)] + X}{1 - \pi}.$$

By substituting (1) into this equation, r can be eliminated. That is,

$$\Gamma(B^*) = \frac{\pi C \cdot F(B^*) + X}{(1 - \pi) + \pi \frac{Q}{B^*}}.$$

From (2), B^* is completely and uniquely determined by Q . Hence, Γ can be expressed in terms of Q :

$$\Gamma(Q) = \frac{\pi C \cdot F(B^*(Q)) + X}{(1 - \pi) + \pi \frac{Q}{B^*(Q)}}.$$

Now we derive the total derivative $\frac{d\Gamma}{dQ}$.

From (2), we can work out
$$\frac{dB^*}{dQ} = \frac{1}{If(B^*)}. \quad (\text{A1})$$

From the expression $\Gamma(B^*, r)$ in (3), we have the total derivative:

$$\begin{aligned} \frac{d\Gamma}{dB^*} &= \frac{\partial\Gamma}{\partial B^*} + \frac{dr}{dB^*} \frac{\partial\Gamma}{\partial r} \\ &= \frac{\partial\Gamma}{\partial B^*} + \frac{d}{dB^*} \left(\frac{P}{B^*} - 1 \right) \frac{\partial\Gamma}{\partial r} \\ &= \frac{\partial\Gamma}{\partial B^*} + \left(\frac{1}{B^*} \frac{dP}{dB^*} - \frac{P}{B^{*2}} \right) \frac{\partial\Gamma}{\partial r}, \end{aligned} \quad (\text{A2})$$

where the equality in the second line uses the optimal lending condition (1). Given the interior region of (3) in which $P = \Gamma$, we can rewrite equation (A2) as

$$\frac{d\Gamma}{dB^*} = \frac{\partial\Gamma}{\partial B^*} + \left(\frac{1}{B^*} \frac{d\Gamma}{dB^*} - \frac{\Gamma}{B^{*2}} \right) \frac{\partial\Gamma}{\partial r}. \quad (\text{A3})$$

Noticing that the term $\frac{d\Gamma}{dB^*}$ appears on both sides of equation (A3), we solve for this term

$$\frac{d\Gamma}{dB^*} = \frac{\frac{\partial\Gamma}{\partial B^*} - \frac{\Gamma}{B^{*2}} \frac{\partial\Gamma}{\partial r}}{1 - \frac{1}{B^*} \frac{\partial\Gamma}{\partial r}}. \quad (\text{A4})$$

Also, from the expression of $\Gamma(B^*, r)$ in (3), we have

$$\begin{aligned} \frac{\partial\Gamma}{\partial B^*} &= \frac{\pi}{1 - \pi} [C - I(1 + r)] f(B^*), \\ \frac{\partial\Gamma}{\partial r} &= \frac{\pi \left\{ \int_0^{B^*} -Bf(B) dB + \int_{B^*}^I (I - B) f(B) dB \right\}}{1 - \pi} = -\frac{\pi}{1 - \pi} Q, \end{aligned}$$

where the second equality follows from the credit market clearing condition (2).

Therefore, we can use these expressions to simplify expression (A4):

$$\frac{d\Gamma}{dB^*} = \frac{[C - I(1 + r)] f(B^*) + (1 + r) \frac{Q}{B^*}}{\frac{1 - \pi}{\pi} + \frac{Q}{B^*}}. \quad (\text{A5})$$

Overall, we have

$$\frac{d\Gamma}{dQ} = \frac{dB^*}{dQ} \frac{d\Gamma}{dB^*} = \frac{[C - I(1+r)]f(B^*) + (1+r)\frac{Q}{B^*}}{If(B^*)\left(\frac{1-\pi}{\pi} + \frac{Q}{B^*}\right)}. \quad (\text{A6})$$

By (A6), we conclude that a sufficient condition for $\frac{d\Gamma}{dQ} > 0$ is $C - I(1+r) > 0$.

The intuition behind Lemma 1 is the following. From (3), $\frac{d\Gamma}{dB^*} = \frac{\pi}{1-\pi}\{[C - I(1+r)]f(B^*) - \frac{dr}{dB^*}Q\}$. Liquidity injections Q enable more entrepreneurs, which are otherwise unable, to make the liquidity investment. Suppose in the economy there is one more entrepreneur switching from non-investing to investing at T_0 . Given r , this would increase the liquidity in the industry at T_1 by an amount $C - I(1+r)$, which corresponds to the NPV of the marginal investment in (4). Further, in the general equilibrium, r changes, which has the effect of the term $\frac{dr}{dB^*}Q$. However, in the general equilibrium, the first term always dominates the second term (i.e., $[C - I(1+r)]f(B^*) - \frac{dr}{dB^*}Q > 0$) when $C - I(1+r) > 0$. In fact, if $\frac{dr}{dB^*}$ is positive, we immediately conclude that Γ must increase in Q because $\Gamma = B^*(1+r)$ by (1) and B^* is increasing in Q . If $\frac{dr}{dB^*}$ is negative, clearly $[C - I(1+r)]f(B^*) - \frac{dr}{dB^*}Q > 0$, meaning $\frac{d\Gamma}{dQ} > 0$. In short, if $C - I(1+r)$ is positive and close to zero, $\frac{dr}{dB^*}$ must be negative; if $\frac{dr}{dB^*}$ is positive, $C - I(1+r)$ must be far above zero and exceed the effect of $\frac{dr}{dB^*}$.

Proof of Proposition 2: For ease of exposition, we reiterate the equilibrium asset price here. That is

$$P = \begin{cases} \mathbb{E}^H(\tilde{x}) & \text{if } \Gamma(B^*, r) > \mathbb{E}^H(\tilde{x}) \\ \Gamma(B^*, r) & \text{if } \Gamma(B^*, r) \in [\mathbb{E}^L(\tilde{x}), \mathbb{E}^H(\tilde{x})], \\ \mathbb{E}^L(\tilde{x}) & \text{if } \Gamma(B^*, r) < \mathbb{E}^L(\tilde{x}) \end{cases}$$

where

$$\Gamma(B^*, r) = \frac{\pi \left[\int_0^{B^*} [C - B(1+r)]f(B)dB + \int_{B^*}^I (1+r)(I-B)f(B)dB \right] + X}{1-\pi}.$$

From Lemma 1, we know that $\Gamma(Q)$ is an increasing function of Q if $C - I(1+r) > 0$. We show that the condition $C - I(1+r) > 0$ holds under a broad range of parameter values. In fact, by $1+r = \frac{\Gamma(B^*, r)}{B^*}$, we have $C - I(1+r) = C - I\frac{\Gamma(B^*, r)}{B^*}$, where B^* is completely determined by Q and r is endogenous. *Ceteris paribus*, if π is sufficiently low, Γ is low and thus $C - I(1+r) > 0$. The numerical example in Appendix A is one case.

We consider the lower bound of the asset price, $\mathbb{E}^L(\tilde{x})$. For our purpose, we choose the parameters to make sure that the upper bound of the asset price is not binding (i.e., the asset price

calculated in $\Gamma(Q)$ is always below $\mathbb{E}^H(\tilde{x})$. We define two cutoffs, \underline{X} and \overline{X} , and divide X into three ranges: $X < \underline{X}$, $X > \overline{X}$, and $\underline{X} \leq X \leq \overline{X}$.

The first range of X is the case in which the asset price is trapped at the lower bound no matter what Q ($\in [0, \overline{Q}]$) is. That is, even if $Q = \overline{Q}$, the asset price calculated in Γ is still (weakly) below $\mathbb{E}^L(\tilde{x})$. Therefore, \underline{X} satisfies $\mathbb{E}^L(\tilde{x}) = \Gamma|_{Q=\overline{Q}, X=\underline{X}}$.

The third range of X is the case in which the asset price is above the lower bound for the whole region of $Q \in [0, \overline{Q}]$. That is, even if $Q = 0$, the asset price calculated in Γ is still (weakly) above $\mathbb{E}^L(\tilde{x})$. Therefore, \overline{X} satisfies $\mathbb{E}^L(\tilde{x}) = \Gamma|_{Q=0, X=\overline{X}}$.

In the second range of $\underline{X} \leq X \leq \overline{X}$, the asset price is the constant $\mathbb{E}^L(\tilde{x})$ when Q is low and then increases in Q when Q is higher.

Proof of Proposition 3: When $X \leq \underline{X}$, the asset price P is constant in Q . Also considering $\frac{dB^*}{dQ} > 0$, we have that when $X \leq \underline{X}$, the equilibrium interest rate $r(Q)$ is strictly decreasing in Q ($\in [0, \overline{Q}]$).

We consider the equilibrium interest rate when $X > \underline{X}$. By $r = \frac{\Gamma}{B^*} - 1$, we have

$$\frac{dr}{dB^*} = \frac{1}{B^*} \frac{d\Gamma}{dB^*} - \frac{\Gamma}{B^{*2}}. \quad (\text{A7})$$

Plugging (A5) into (A7), we have

$$\frac{dr}{dB^*} = \frac{[C - I(1+r)] f(B^*) B^* - \frac{1-\pi}{\pi} \Gamma}{\left(\frac{1-\pi}{\pi} B^* + Q\right) B^*}. \quad (\text{A8})$$

By $1+r = \frac{P(Q)}{B^*(Q)}$, considering $P = \Gamma$, we have

$$\begin{aligned} \frac{dr}{dQ} &= \frac{dB^*}{dQ} \frac{dr}{dB^*} \\ &= \frac{1}{I f(B^*)} \frac{[C - I(1+r)] f(B^*) B^* - \frac{1-\pi}{\pi} \Gamma}{\left(\frac{1-\pi}{\pi} B^* + Q\right) B^*} \\ &= \frac{[C - I(1+r)] B^* - \frac{1-\pi}{\pi} \frac{\Gamma}{f(B^*)}}{\left(\frac{1-\pi}{\pi} B^* + Q\right) B^* I}. \end{aligned} \quad (\text{A9})$$

Hence, $\frac{dr}{dQ} > 0$ if and only if $f(B^*) > \frac{1-\pi}{\pi} \left[\frac{C}{1+r} - I \right]^{-1}$ by noting that $\frac{\Gamma}{B^*} = 1+r$. In equilibrium, we guarantee that $C > I(1+r)$, so the right-hand side of this condition is bounded. Note that by the proof of Lemma 1, if $\frac{dr}{dB^*}$ is positive, $C - I(1+r)$ must be greater than a positive number. When $f(B^*)$ is sufficiently high, $\frac{dr}{dQ}$ is positive. The numerical example in Appendix A illustrates these.

We also consider the special case of $\frac{P(Q)}{B^*(Q)}|_{Q=0}$. In this case, B^* solves $\int_{B^*}^I (I - B) f(B) dB = \int_0^{B^*} B f(B) dB$ by noting that B^* has a unique solution based on (2'), and $\Gamma = \frac{\pi C \cdot F(B^*) + X}{1-\pi}$.

Hence, r is determined. We can also work out $\frac{dr}{dQ}$ at $Q = 0$ by (A9). That is, $\frac{dr}{dQ}|_{Q=0} = \frac{CB^* - [\pi C \cdot F(B^*) + X] \left(\frac{I}{1-\pi} + \frac{1}{\pi f(B^*)} \right)}{\left(\frac{1-\pi}{\pi} B^* + Q \right) B^* I}$. *Ceteris paribus*, if π is sufficiently low, $\frac{dr}{dQ}|_{Q=0} < 0$.

To summarize, when $X > \underline{X}$, we can choose some function $f(B)$ (i.e., $f(B)$ is sufficiently high in some region of B) and some \bar{Q} such that r decreases first and then increases in Q within the interval $Q \in [0, \bar{Q}]$.

Proof of Proposition 4: First, given a Q_i for each sector, solve for the equilibrium within each sector, that is, solve for the triplet $\{B_i^*, P_i, r_i\}$. In particular, we obtain the function $r_i(Q_i)$. If conditions in Proposition 3 are satisfied, $r_1(Q_1)$ is a decreasing function and $r_2(Q_2)$ is a ‘U’-shaped function. Second, by considering the link between the two sectors, (5a) and (5e), we can work out Q_i (for $i = 1$ and 2) for a given Q . That is, by considering $r_1(Q_1) = r_2(Q_2) = r$ and $Q_1 + Q_2 = Q$, we obtain the unique Q_1 and Q_2 , and r . In fact, by aggregating $r_1(Q_1)$ and $r_2(Q_2)$, we can obtain an ‘aggregate’ function $r(Q)$. That is, for a given r , we find the corresponding Q_1 which solves $r_1(Q_1) = r$ and Q_2 which solves $r_2(Q_2) = r$, and then aggregate Q_1 and Q_2 as $Q = Q_1 + Q_2$. Under a wide set of chosen parameters, the ‘aggregate’ function $r(Q)$ is a ‘U’-shaped function. The numerical example above in Appendix A is one case. Thus, for a given Q , we have unique Q_1 , Q_2 and r .

Proof of Proposition 5: From the proof of Proposition 4, we have that the ‘aggregate’ function $r(Q)$ is a ‘U’-shaped function (i.e., decreasing first and then increasing). By Proposition 3, $r_1(Q_1)$ is a decreasing function. Therefore, to maximize Q_1 , we need to choose a Q to minimize $r(Q)$. Clearly, there is a unique Q that minimizes r and thus maximizes Q_1 .

Proof of Proposition 6: Considering that $r_1(Q_1)$ is a decreasing function and $r_2(Q_2)$ is a ‘U’-shaped function, we find a unique Q_1^* that solves $r_1(Q_1^*) = r_{\min}$ and a unique Q_2^* that solves $r_2(Q_2^*) = r_{\min}$. We define $Q^* = Q_1^* + Q_2^*$. By $r_1(Q_1) = r_2(Q_2) = r$ and $Q_1 + Q_2 = Q$, we have that Q_1 is increasing in Q when $Q < Q^*$ and decreasing in Q when $Q > Q^*$, and that Q_2 is increasing in Q .

Proof of Proposition 7: Given Q_i and C_i for each sector, solve for the equilibrium within each sector, that is, solve for the triplet $\{B_i^*, P_i, r_i\}$. Based on the proofs of Propositions 2 and 3, the cash flow C affects the results in Propositions 2 and 3 only quantitatively, not qualitatively. Therefore, for some C_1 and C_2 , $P_1(Q_1)$ and $P_2(Q_2)$ have the properties in Proposition 2, and $r_1(Q_1)$ and $r_2(Q_2)$ have the properties in Proposition 3.

In addition, we need that the asset price in Sector 1 is binding at $E^L(\tilde{x})$ and that in Sector 2

is not. Note that $C_1 > C_2$ and $X_1 < X_2$. When π is chosen to be sufficiently small, the first term in the numerator of (3) which involves C has a limited role in determining the asset price while the second term which is about X becomes crucial. Hence, when the gap between X_1 and X_2 is sufficiently big, the result is obtained.

Overall, the conditions to guarantee that Program 1 has a unique interior optimal Q ($\in (0, \bar{Q})$) are that π is sufficiently small and $f(B)$ is sufficiently high in some region of B .

A numerical example is given to illustrate the existence of relevant parameters. We continue the previous simulation exercise in Appendix A. Let $I = 1$, $\pi = 0.185$, $\mathbb{E}^H(\tilde{x}) = 1.2$, and $\mathbb{E}^L(\tilde{x}) = 1.1$. The distribution of B is $f(B) = \log \frac{1}{1-B}$ for $B \in [0, 1)$. As the maximum total amount of liquidity that any one sector can demand is $Q^{\max} = \int_0^1 Bf(B)dB = 0.75$, we set $\bar{Q} = 2Q^{\max} = 1.5$. We choose $C_1 = 5.6 > C_2 = 4$ and $X_1 = 0.01 < X_2 = 0.4$. We find that $P_1(Q_1) = 1.1$ for any $Q_1 \in [0, 0.75]$ and that when Q_2 is sufficiently high $P_2(Q_2)$ is above 1.1 and increasing in Q_2 . The optimal level of liquidity injection to maximize Q_1 is $Q^* = 1.1787$, which lies within $(0, \bar{Q})$, where $\bar{Q} = 1.5$.

Proof of Proposition 8: First, we show that W is divided among investing entrepreneurs, non-investing entrepreneurs, and the government. For simplicity, we first consider the one-sector economy. By using (2) and (2'), we have

$$\begin{aligned} W &= \int_0^{B^*} (C - I)f(B) dB \\ &= \underbrace{\int_0^{B^*} [C - (1+r)B - (I-B)]f(B) dB}_{\text{Surplus for investing firms}} + \underbrace{r \int_{B^*}^I (I-B)f(B) dB}_{\text{Surplus for non-investing firms}} + \underbrace{rQ}_{\text{Surplus for the government}}. \end{aligned}$$

For the two-sector economy, it is easy to show:

$$\begin{aligned} W &= \sum_i \int_0^{B_i^*} (C_i - I)f(B) dB \\ &= \underbrace{\sum_i \int_0^{B_i^*} [C_i - (I-B) - (1+r)B]f(B) dB}_{\text{Surplus for investing firms}} + \underbrace{\sum_i r \int_{B_i^*}^I (I-B)f(B) dB}_{\text{Surplus for non-investing firms}} + \underbrace{rQ}_{\text{Surplus for the government}}. \end{aligned}$$

Second, by using (2'), it is easy to show that W can be rewritten as

$$W = \left[Q + 2 \int_0^I (I-B)f(B)dB \right] \cdot \bar{R} - 2 \int_0^I (I-B)f(B)dB - Q.$$

The first-order condition of W with respect to Q is

$$\bar{R}(Q) + \left[Q + 2 \int_0^I (I - B)f(B)dB \right] \frac{d\bar{R}(Q)}{dQ} = 1. \quad (\text{A10})$$

Clearly, $\bar{R}(Q)$ is decreasing in Q for $Q > Q^*$ due to the crowding-out effect (i.e., $F(B_1^*)$ is decreasing and $F(B_2^*)$ is increasing in Q). Under some parameter values, the LHS of (A10) overall is a decreasing function of Q for some range of Q , and there is a unique solution to (A10).

To obtain further insight, we can rewrite W as

$$W = \left[\underbrace{C_2 \cdot \begin{pmatrix} F(B_1^*(Q_1)) + F(B_2^*(Q_2)) \\ -F(B_1^*(Q_1^*)) - F(B_2^*(Q_2^*)) \end{pmatrix}}_{\text{additional injected liquidity entering Sector 2}} - \underbrace{(C_1 - C_2) \cdot (F(B_1^*(Q_1^*)) - F(B_1^*(Q_1)))}_{\text{some liquidity moving out of Sector 1 to Sector 2}} \right] + \left[\begin{matrix} C_1 \cdot F(B_1^*(Q_1^*)) \\ +C_2 \cdot F(B_2^*(Q_2^*)) \end{matrix} \right] - 2 \int_0^I (I - B)f(B)dB - Q,$$

where Q_i^* is the net liquidity inflow for Sector i when $Q = Q^*$ in Proposition 7. By $2 \int_0^I (I - B)f(B)dB + Q = I \cdot (F(B_1^*) + F(B_2^*))$, we have that $F(B_1^*(Q_1)) + F(B_2^*(Q_2))$ is increasing in Q . Also, Q_1 and hence $F(B_1^*(Q_1))$ are decreasing in Q . That is, an increase in Q has two opposite forces on income at T_1 : additional liquidity injected entering Sector 2 increases income, but more liquidity injected also crowds more liquidity out from Sector 1 to Sector 2, decreasing income because $C_1 > C_2$. Under certain conditions, the second force dominates the first force when Q is too high, and, therefore, there exists an optimal Q .

We continue the numerical example in the proof of Proposition 7. The optimal level of liquidity injection that maximizes W is coincidentally also $Q = 1.1787$ in Proposition 8. That is, any additional liquidity injection beyond Q^* (the optimal level to maximize B_1^*) will reduce the total surplus.

Proof in Section 5.1: Denote by α the proportion of entrepreneurs from Sector 1 who decide to trade in asset market 2. We now explore three alternative scenarios regarding agents' beliefs, under which we draw the same conclusion.

1) Beliefs of an entrepreneur are perfectly correlated in the two asset markets.

In this scenario, an entrepreneur who has high beliefs in asset market 1 necessarily also has high beliefs in asset market 2, so $\alpha = 0$. This is because from the perspective of high-beliefs entrepreneurs, asset market 1 is more undervalued and thus has a higher return to speculation than asset market 2 (i.e., $\frac{\mathbb{E}^H(\tilde{x})}{P_1} > \frac{\mathbb{E}^H(\tilde{x})}{P_2}$ by $P_1 < P_2$). So high-beliefs speculators in Sector 1 stay

and focus on trading in Sector 1, rather than migrate and divert funds to trade in asset market 2. Because $\alpha = 0$, the results in the main model do not change.

2) Beliefs of an entrepreneur are not perfectly correlated in the two asset markets.

In this scenario, $\alpha > 0$. Entrepreneurs from Sector 1 with low beliefs in market 1 but high beliefs in market 2 participate in trading in market 2. Then, the asset price Γ_1 of Sector 1, given in (3), remains the same. This is because neither the sellers nor the buyers in market 1 change, by noting that those who migrate to trade in Sector 2 must be among low-beliefs sellers in market 1.

However, the asset price Γ_2 of Sector 2 changes and becomes

$$\Gamma_2 = \frac{\pi [C_2 \cdot F(B_2^*) - Q_2(1+r)] + \alpha \{[C_1 \cdot F(B_1^*) - Q_1(1+r)] + P_1\} + X_2}{1 - \pi}. \quad (\text{A11})$$

Compared with Γ in (3), there is an extra term in the numerator of Γ_2 in (A11): entrepreneurs from Sector 1 use their cash income from projects or deposits (i.e., the term $C \cdot F(B_1^*) - Q_1(1+r)$) as well as their income from selling assets in market 1 (i.e., the term P_1) to purchase assets in market 2. We can prove that Γ_2 in (A11) still has the same properties as in Propositions 2 and 3, and hence the conclusions of the model do not change. Concretely, we first show that Γ_2 is increasing in Q . We discuss two ranges of Q : $\frac{dr}{dQ} < 0$ (the range of the allocation effect) and $\frac{dr}{dQ} > 0$ (the range of the crowing-out effect). For the first range, both Q_1 and Q_2 are increasing in Q . By the proof of Lemma 1, under the sufficient condition of $C_i - I(1+r) \geq 0$, we have that $C_i \cdot F(B_i^*) - Q_i(1+r)$ is increasing in Q . Also, P_1 is a constant with $P_1 = \mathbb{E}^L(\tilde{x})$. Hence, Γ_2 is increasing in Q . For the second range, because $r = \frac{\Gamma_2}{B_2^*} - 1$ and B_2^* is increasing in Q , we have that Γ_2 is increasing in Q . For the curve of $r_2(Q_2)$ to be U-shaped, as shown in the Proof of Proposition 3, $f(B)$ needs to be sufficiently high in some region of B .

3) There exist no heterogeneous beliefs among entrepreneurs from Sector 1.

In this scenario, we keep the setup for Sector 2 the same as in the main model but assume a slightly different setup for Sector 1. It is alternatively assumed that entrepreneurs do not have heterogeneous beliefs about a project in Sector 1. In other words, a project's valuation at T_1 is common knowledge among entrepreneurs for Sector 1. This setup essentially implies that there is *no speculative trade* in Sector 1 while there is in Sector 2; in other words, entrepreneurs have no disagreement regarding a project's valuation (i.e., on cash flow \tilde{x}) at T_1 in the real sector.

To model this setup without introducing new notations, simply set $\pi = 0$ for Sector 1; that is, all entrepreneurs in Sector 1 have common beliefs: $\Pr[\tilde{x} = u] = \theta_L$.²⁵ Then, it is easy to show that P_1 is a constant and does not change with the liquidity injection, i.e., $P_1 = \mathbb{E}^L(\tilde{x})$.²⁶ As in scenario

²⁵To save notations, the probability under the common belief is written as θ_L .

²⁶For simplicity, we assume that α is small, so that the cash-in-the-market price in market 1, induced by the selling from those "liquidity" traders who have "better" investment opportunities in the other market, will not drop below the asset's fundamental value.

2), entrepreneurs of Sector 1 may have high beliefs in market 2, and so they participate in buying assets in market 2. That is, the asset price Γ_2 of Sector 2 is given by (A11). Then, it is easy to show that the model results under scenario 3) are the same as those under scenario 2).

Proof in Section 5.3: It is easy to obtain the result based on Figure 5. When Q is low such that $r_2(Q_2)$ is decreasing in Q_2 , r_2 is decreasing in Q ; by $r_1 - r_2 = \Delta r$, in equilibrium r_1 is also decreasing in Q , which means that Q_1 is increasing in Q . When Q is high such that $r_2(Q_2)$ is increasing in Q_2 , r_2 is increasing in Q ; by $r_1 - r_2 = \Delta r$, in equilibrium r_1 is also increasing in Q , which means that Q_1 is decreasing in Q . In short, the model results do not change qualitatively under the new assumption of $r_1 - r_2 = \Delta r$.

Details for Section 5.5: We construct the equilibrium in which $r_1(Q_1)$ is increasing in Q_1 (in some region of Q_1) and $r_2(Q_2)$ is decreasing in Q_2 (in some region of Q_2). The following is one case. By $1+r = \frac{P(Q)}{B^*(Q)}$, if P increases faster than B^* , r is increasing in Q ; otherwise, r is decreasing in Q . For Sector 2, if X_2 is sufficiently high such that Γ_2 (i.e., Γ in (3) for Sector 2) is binding at the upper bound $\mathbb{E}^H(\tilde{x})$, then clearly $r_2(Q_2)$ is decreasing in Q_2 . For Sector 1, if X_1 is sufficiently lower such that Γ_1 (i.e., Γ in (3) for Sector 1) is lower than and not binding at $\mathbb{E}^H(\tilde{x})$, then based on the proof of Proposition 3, we can choose some function $f(B)$ (i.e., $f(B)$ is sufficiently high in some region of B) such that $r_1(Q_1)$ is increasing in Q_1 . Furthermore, we need that the ‘aggregate’ function $r(Q)$ is decreasing in Q , where $r(Q)$ is defined in the proof of Proposition 4; that is, the price (interest rate) elasticity of demand to liquidity for Sector 2 is higher than that for Sector 1. To achieve this, it is realistic to assume the two sectors are heterogeneous in cash flow C and asset specificity X as well as in cash flow \tilde{x} and distribution $f(B)$.

A numerical example is to illustrate the above equilibrium. Let $I = 1$ and $\pi = 0.4$. For Sector 1, the distribution of B is a truncated normal with $f(B) = \frac{100}{\Phi(0.508) - \Phi(0)} \varphi\left(\frac{B-0.5}{0.01}\right)$ in the support $[0, 0.508]$, where $\varphi(\cdot)$ stands for the p.d.f. of the standard normal, and $C_1 = 1.8$, $X_1 = 0.15$, $\mathbb{E}^H(\tilde{x}_1) > 0.8781$, and $\mathbb{E}^L(\tilde{x}_1) < 0.8544$, where \tilde{x}_i denotes the stochastic cash flow \tilde{x} at T_2 for Sector $i = 1$ and 2. For Sector 2, the distribution of B is uniform within the support $[0.5007, 0.5037]$, and $C_2 = 1.795$, $X_2 = 0.163$, and $\mathbb{E}^L(\tilde{x}_2) < \mathbb{E}^H(\tilde{x}_2) = 0.8671$. Denote by Q_i^{\max} the maximum liquidity demand for Sector $i = 1$ and 2. Then, we find out that $P_1(Q_1)$ is monotonically increasing in $Q_1 \in [0, Q_1^{\max}]$ with $Q_1^{\max} = 0.4963$, while $P_2(Q_2)$ is binding at its asset price upper bound $\mathbb{E}^H(\tilde{x}_2) = 0.8671$ for all $Q_2 \in [0, Q_2^{\max}]$ with $Q_2^{\max} = 0.5022$. Figure B1 depicts $r_1(Q_1)$ and $r_2(Q_2)$.

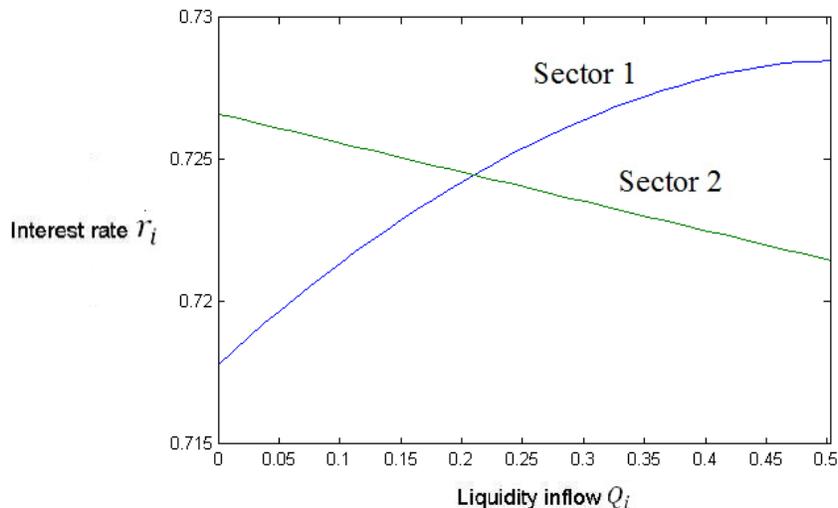


Figure B1: A numerical example of crowding-out with a falling interest rate

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